Geometry and Entanglement in 2d CFT

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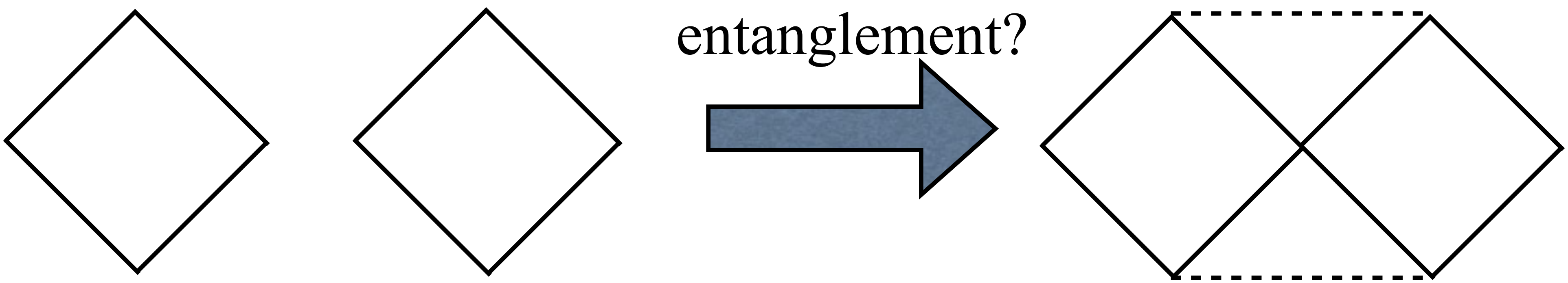
In quantum gravity, spacetime geometry emerges from some underlying d.o.f.

We do not really understand how this works, but entanglement plays a key role:



[Maldacena; Ryu & Takayanagi;
Van Raamsdonk; Maldacena and Susskind; etc.]

In particular, in AdS/CFT

$$\textit{Entanglement entropy in CFT} = \frac{\text{Area}}{4} \textit{ in gravity.}$$

Ryu & Takayanagi '06
Casini, Huerta, Myers '12
Lewkowycz & Maldacena '13

This follows from the rules of AdS/CFT, but is still mysterious.

- Why does this happen in QFT?
- To what universality class of QFTs does this apply?
- Can we systematically correct this result?

I will give some partial answers to these questions in 1+1D conformal field theories.

Tools

- Operator-product expansion (OPE)
- Large-central-charge expansion

Results

- Universal contribution to entanglement entropy in a class of excited states in 1+1D, strongly interacting, non-rational CFTs
- Directly related to emergent 3d geometries
- *(Tools for universality at large central charge: "1/c expansion")*

based mostly on:
TH '13
Asplund, Bernamonti, Galli, TH '14

Large central charge expansion

Consider a 1+1D CFT with:

- central charge $c \gg 1$ (“large N”)

- sparse spectrum of low-dimension operators

$$eg, \; N_{states}(\Delta < \Delta_*) = \text{finite as } c \to \infty$$

In 1+1 dimensions:

Space is a line or circle. Choose a finite interval **A:**

Entanglement entropy

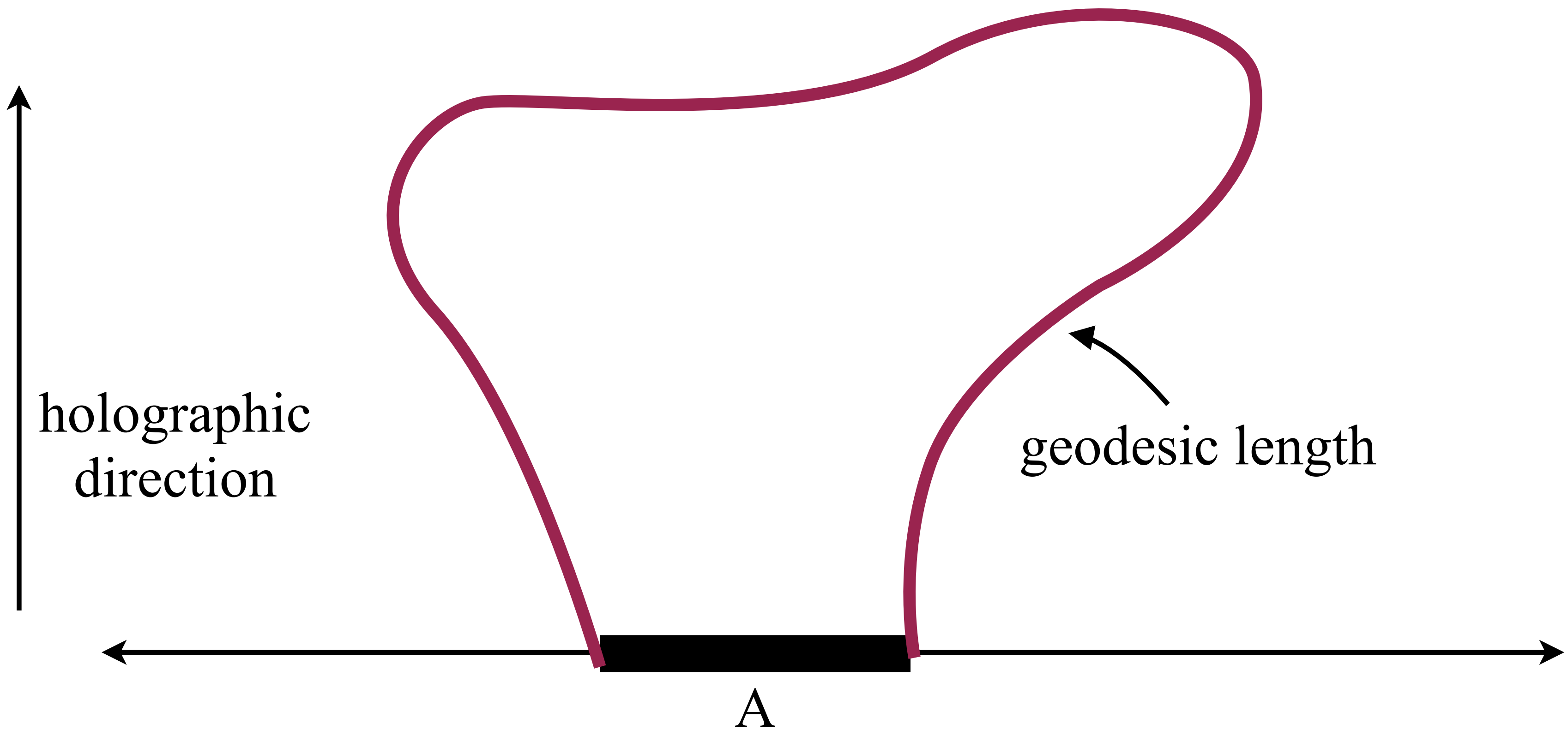
$$S_A = -\text{tr}\ \rho_A \log \rho_A$$

Goal:

Compute entanglement entropy for the full system in a *highly excited pure state*.

TH '13;
Asplund, Bernamonti, Galli, TH '14

AdS/CFT suggests a simple, universal answer, independent of the other details of the CFT:

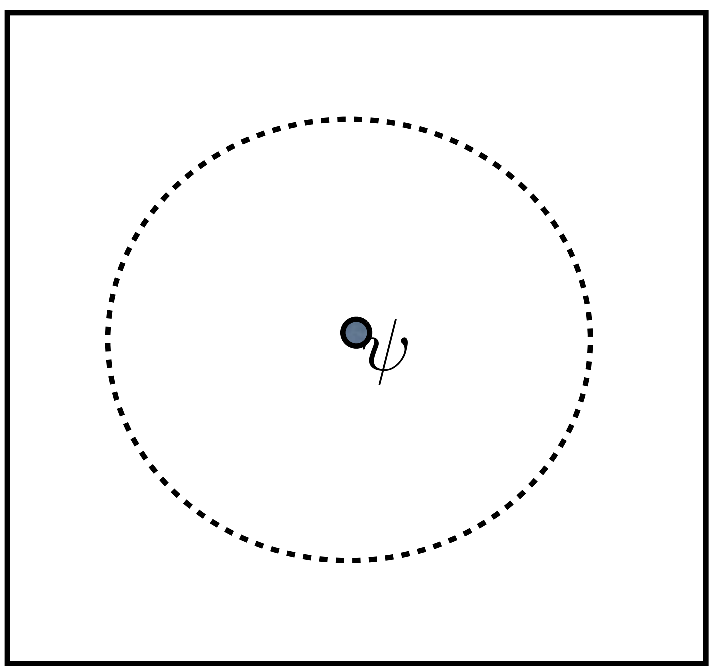


Ryu & Takayanagi;
Hubeny, Rangamani, Takayanagi

The state we consider is a conformal primary state on a circle.

In radial quantization, this is created by inserting a primary operator at the origin:

$$|\psi\rangle = \psi(0)|0\rangle$$



To ensure finite "temperature" in large-c limit, scale

$$\Delta_\psi = O(c)$$

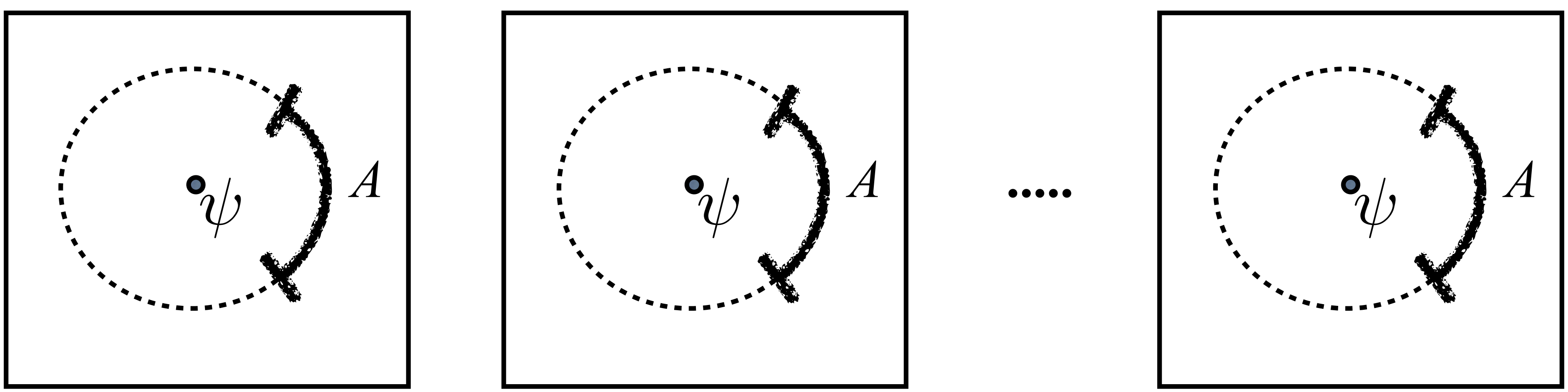
To compute EE, use the replica method:

Calabrese & Cardy; etc

$$Z_n = \mathrm{Tr}\ \rho_A^n$$

$$S_A = \frac{1}{1-n} \log Z_n \Big|_{n \to 1}$$

$$\mathrm{Tr}\,\rho_A^n =$$



This many-sheeted path integral can be recast as a correlation function on a single sheet,

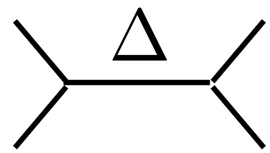
$$\begin{aligned} \mathrm{Tr}\,\rho_A^n &= \langle\psi|\sigma\sigma|\psi\rangle \\ &= \langle 0|\psi\sigma\sigma\psi|0\rangle \end{aligned}$$

σ is a `twist operator' that glues the sheets together.

This is useful because there is a lot of technology to compute 4-point correlators in CFT.

They must have a conformal block expansion:

$$\langle \psi \sigma \sigma \psi \rangle = \sum_{\Delta} c_{\psi\psi\Delta} c_{\sigma\sigma\Delta}$$



$$= \sum_{\Delta} c_{\psi\psi\Delta} c_{\sigma\sigma\Delta} |\mathcal{F}(\Delta, H_n, z)|^2$$

Virasoro Conformal Block

OPE coefficients

$$H_n = \frac{c}{24}(n - 1/n) = \text{ dimension of twist operator}$$

First applied in this context by Headrick ’10

In general, this is not universal, and it is hopeless to calculate.

But for central charge $c \to \infty$, the Virasoro block simplifies,

$$\mathcal{F} \approx e^{cf}$$

and we expect the vacuum representation to dominate the sum:

$$\langle \psi \sigma \sigma \psi \rangle \approx e^{2cf_{\mathrm{vacuum}}}$$

This term includes the contribution from all operators constructed from the stress tensor:

$$1, \quad T, \quad \partial T, \quad T\partial T, \quad T^2, \quad \ldots$$

This sector of the CFT is dual to the graviton sector of 3d gravity. We'll see it exactly reproduces the universal gravity results.

Assumptions & Caveats

- Large-*c*
- "Sparseness" --> leading order in 1/c, but all orders in the OPE
- However we have given precise criteria and proved this only for a special setup (two intervals in vacuum, n<=2)

TH '13; TH, Stoica, Keller '14

We still need to calculate *f*

$$\mathcal{F} \approx e^{cf}$$

Briefly:

- [Zamolodchikov, 80s] Null decoupling equations in Liouville can be used to compute *f* by finding flat SL(2) connections on the CFT manifold. BPZ
- Certain *special* flat SL(2) connections correspond to 3d hyperbolic geometries.
- The vacuum block calculation corresponds exactly to one of these special SL(2) connections that is realized in 3d geometry.
- This reduces the CFT calculation to the calculation of geodesics in a 3-dimensional spacetime!

Faulkner ’13. TH ’13.

Exactly the block we need was already calculated (for related reasons) by Fitzpatrick, Kaplan, and Walters!

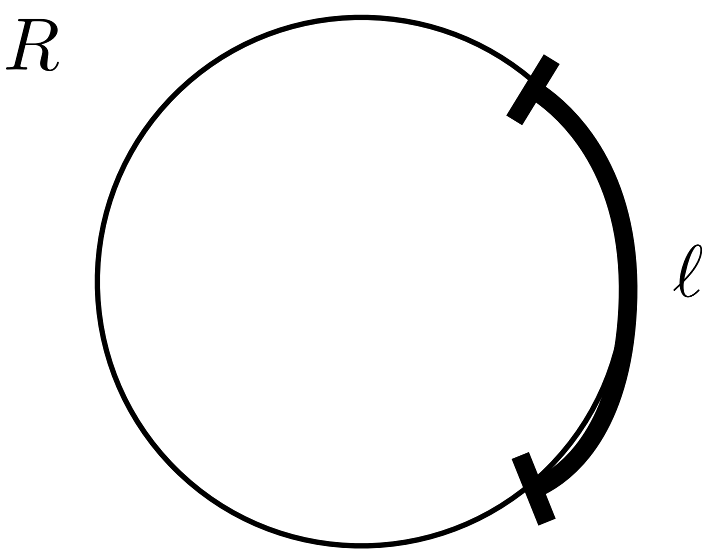
This gives a closed-form answer for the entanglement entropy in an excited state.

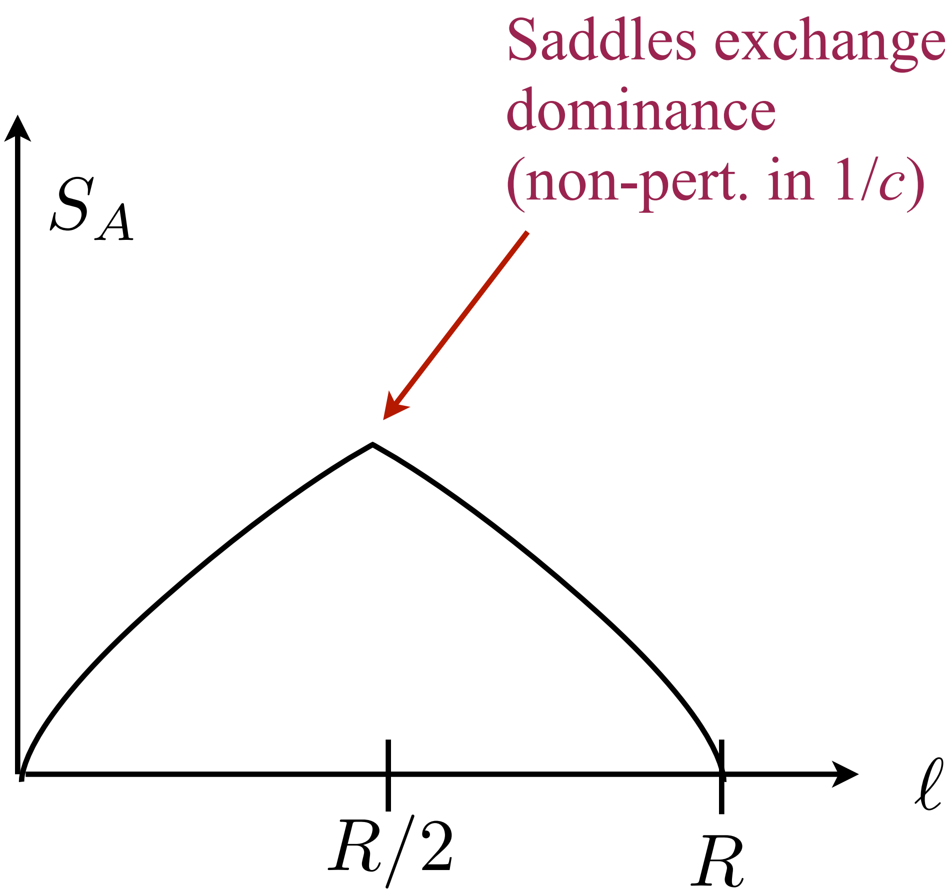
In the limit n--> 1, the block is exactly equal to the length of a geodesic on a 3d defect geometry.

I will discuss the physics of this answer in two situations:

- Energy eigenstates
- "Local quench" experiment

Energy Eigenstates on a Circle

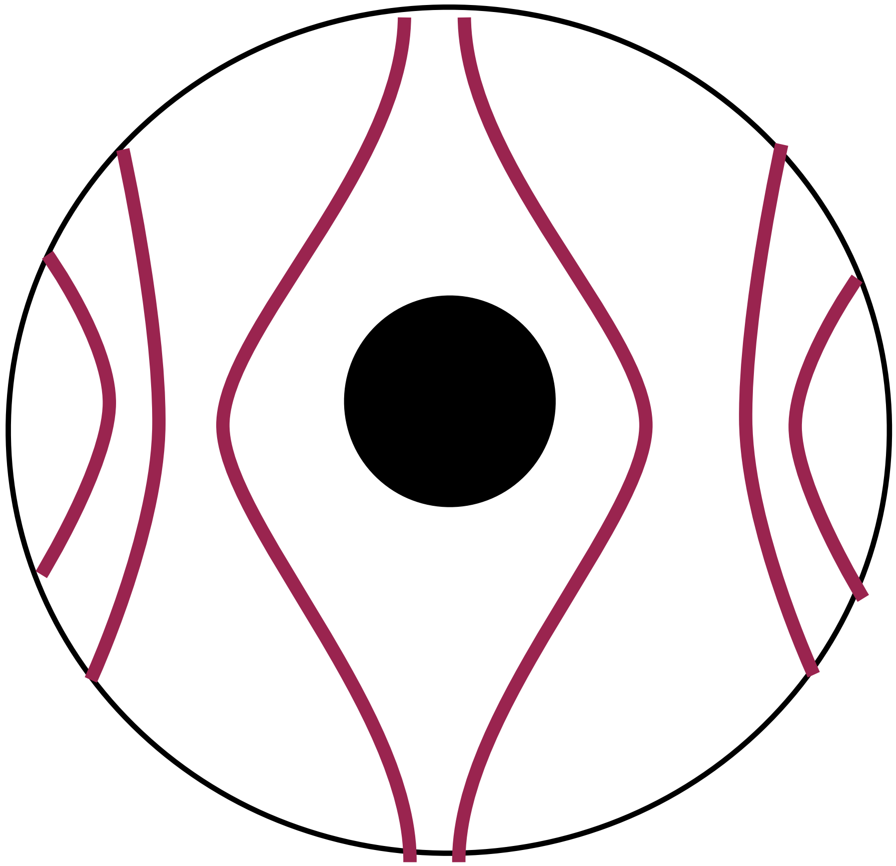




$$S_A = \frac{c}{3} \log\left[\frac{\beta_\psi}{\pi} \sinh\left(\frac{\pi \ell}{\beta_\psi}\right)\right] \qquad \beta_\psi \equiv \frac{2\pi}{\sqrt{24h_\psi/c - 1}}$$

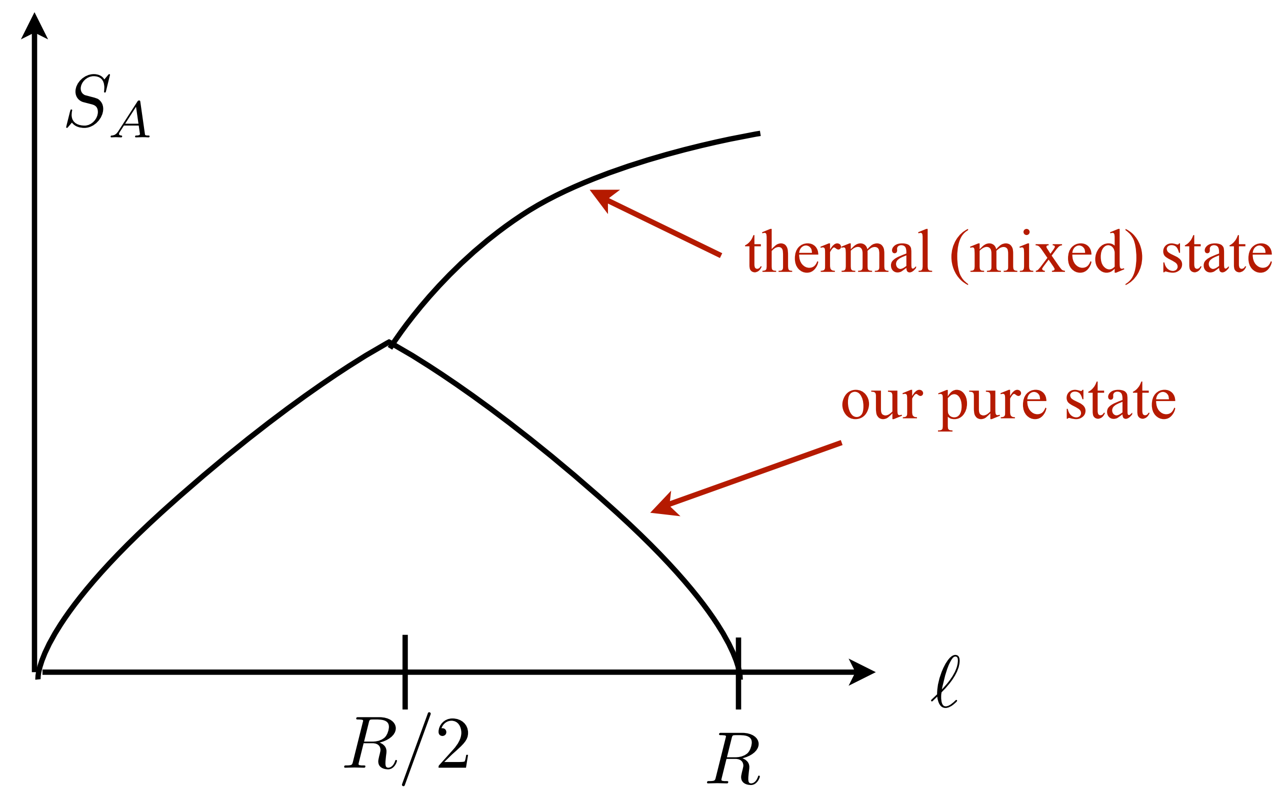
Holographic dual

Black hole in global AdS



conformal block
=
geodesic length

Comments on thermalization



compare: Page theorem

Does $l < R/2$ answer always thermalize?

Unknown. One situation where we might fail is if the state $|\psi\rangle$ has light operators with large expectation values

$$\langle\psi|O_{light}|\psi\rangle = O(c)$$

These large OPE coefficients could affect the saddlepoint analysis.

Example: T=0 BPS microstates Giusto and Russo '14

This directly relates eigenstate thermalization (on a circle) to black hole "hair" / "fuzzballs".

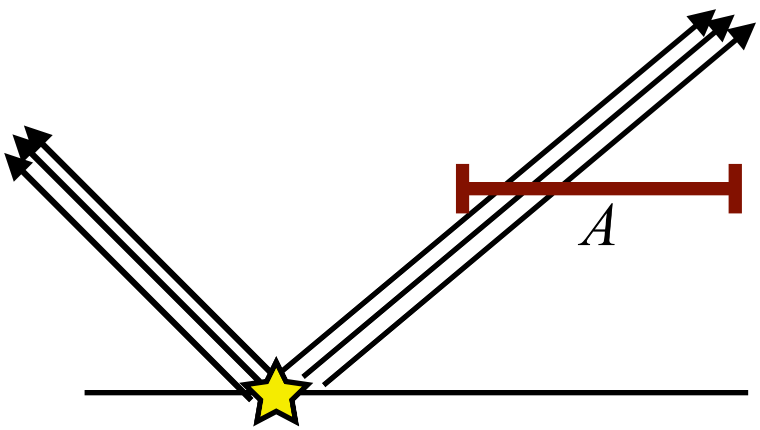
Application #2: "Local quench experiment"

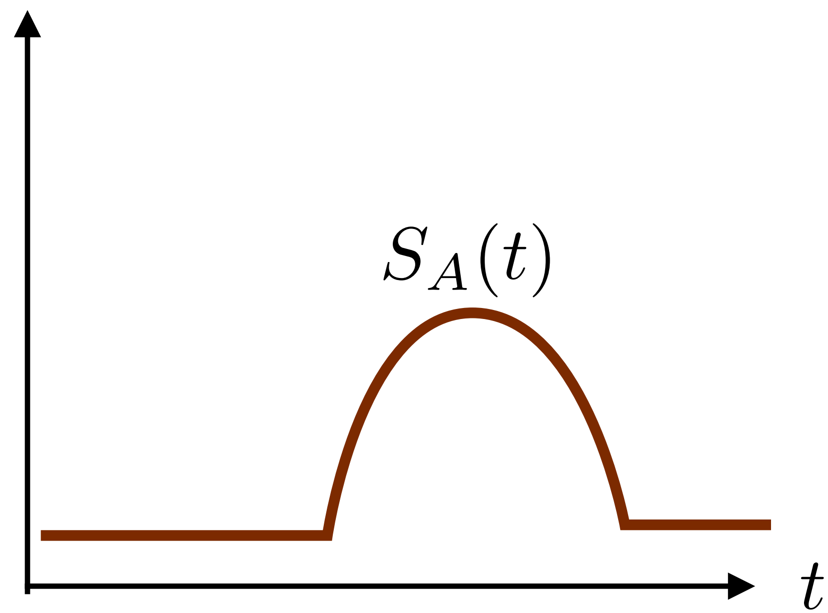
$$|\psi\rangle = \psi(i\epsilon)|0\rangle$$

$$\epsilon \to 0$$

Nozaki, Numasawa, Takayanagi '14;
Caputa, Nozaki, Takayanagi '14

Lorentzian cartoon





This state is related to our previous one by a conformal transformation.

But conformally mapping into the Lorentzian regime introduces a subtlety: operator ordering

$$\langle\psi|\sigma\sigma|\psi\rangle$$

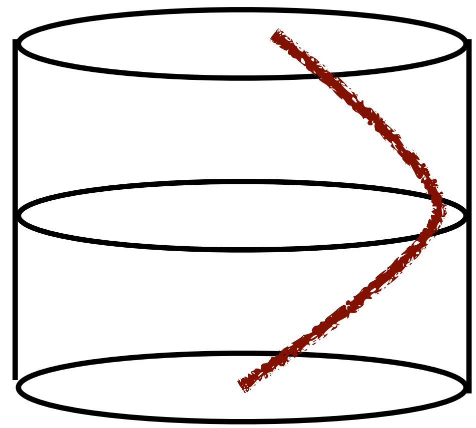
When the dust settles, the height of the "bump" in EE is computed by a braiding of the Virasoro vacuum block,

$$\mathcal{F}(ze^{2\pi i}), \qquad z \to 0$$

(This is very roughly a non-rational cousin of the *quantum dimension* in rational CFT.) cf: Nozaki, Numasawa, Takayanagi '14;

Comparison to holography

- A holographic model of a local quench was proposed by Nozaki, Numasawa, and Takayangi '13: An infalling particle geometry



- This is the holographic dual of our calculation. The infalling particle "hits" the boundary at imaginary time $t = i\epsilon$

- CFT results agree precisely with geodesic lengths on this background.

Conclusions

Large c expansion

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Sparse spectrum of low-dimension operators

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universality and 3d gravity

Some questions

- $1/c$ corrections?
- Including matter?
- Higher dimensions?