Scaling theory of the cuprate strange metals

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- The ‘strange metal’ regime of the cuprates exhibits multiple unconventional scaling behaviors.
- This regime cannot be described by long lived quasiparticles:
(i) The resistivity is too large (‘bad metals’):

$$\ell_{\mathrm{mfp}} \sim v_F \tau \lesssim \ell_{dB}$$

(ii) The ‘Drude peaks’ are too broad:

$$\Gamma \sim k_B T$$

- Absence of quasiparticles means that doing 'textbook' computations with Feynman diagrams, Boltzmann equations, etc. is likely to be problematic.

- Question for this talk:

  To what extent can a single set of 'quantum critical' degrees of freedom produce the observed scaling laws?

- This talk is about charting out systematically the space of possible non-quasiparticle kinematics, starting with the simplest case.

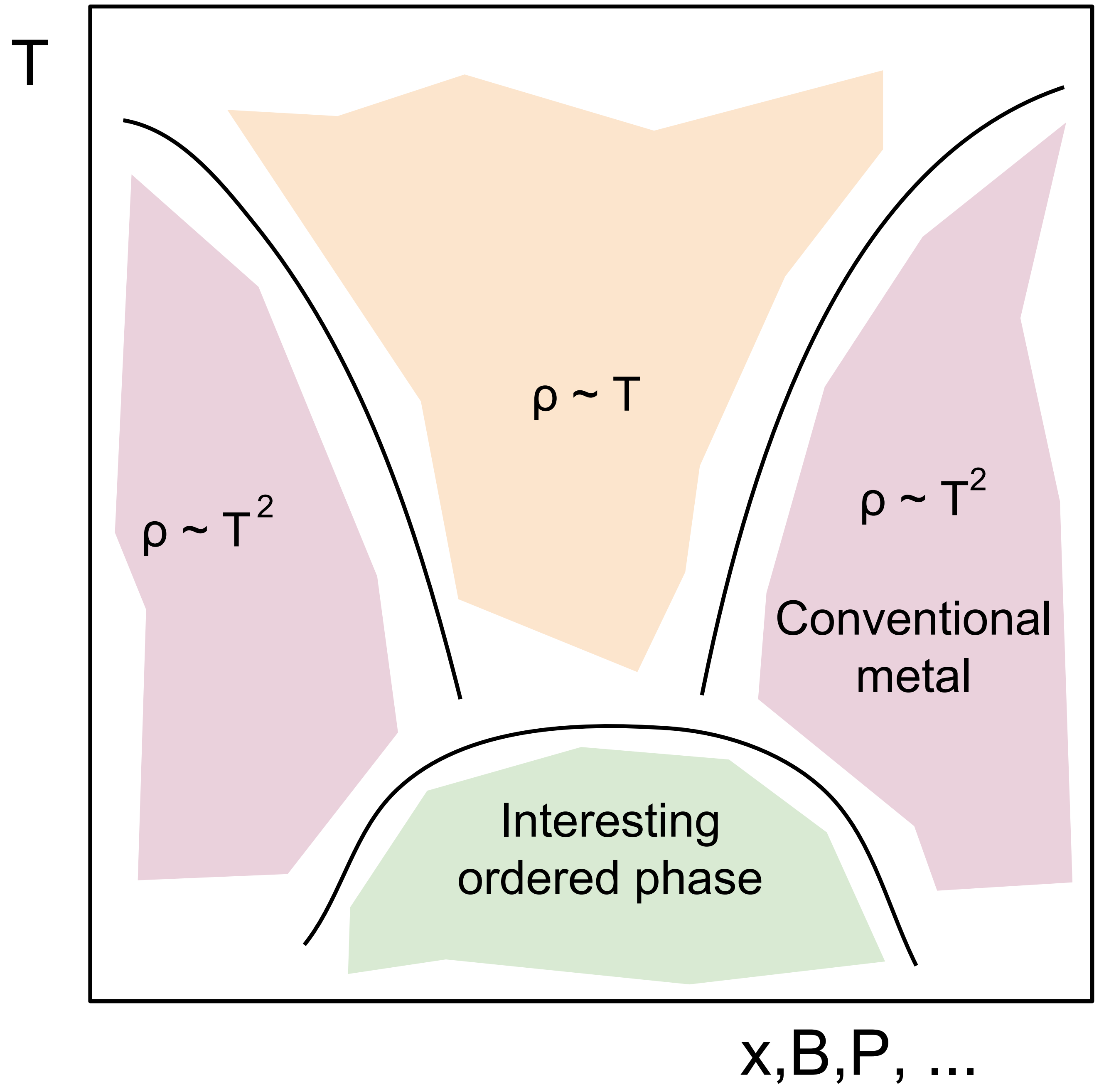
- **Take home messages:**

  (i) It pays to be open minded about what quantities can get anomalous dimensions.

  (ii) Simplest scaling works better than it needed to (get out quite a bit more than we put in). Many predictions.

  (iii) Some (thermodynamic) quantities don't work so well. Either transport dominated by a subset of degrees of freedom or need to go to next simplest case.

Cartoon phase diagram



- Objective: describe the strange metal by dimensional analysis, with the only scale being the temperature T.

- This quantum critical regime is unstable at Tc, i.e. spontaneously generates a scale.

- Interested in high temperature regime away from this scale, e.g. $2T_c \lesssim T$.

The three exponents

- **z: dynamical critical exponent**

$$\xi \sim \frac{1}{T^{1/z}}$$

- **θ: hyperscaling violation exponent**

$$f \sim T \cdot T^{(2-\theta)/z}$$

- **Φ: anomalous dimension for charge**

$$n \sim T^{(2-\theta+\Phi)/z}$$

Comment on Φ

- Commonly argued that the charge density operator cannot get an anomalous dimension Φ.
- This is certainly true in a CFT, but not in general.
- Some of these arguments also apply to hyperscaling violation θ and are therefore falsified by Fermi liquid theory.
- There exist holographic models with nonzero Φ.

Comment on Φ

- The above said, I do not know ‘simple’ weakly coupled field theory models, with conventional scaling towards k=0, with nonzero θ and/or Φ. Would be nice to find some.

Transport

- **The three exponents determine the scaling of transport quantities:**

$$\begin{pmatrix} j \\ j^Q \end{pmatrix} = \begin{pmatrix} \sigma & T\alpha \\ T\alpha & T\overline{\kappa} \end{pmatrix} \begin{pmatrix} E \\ -(\nabla T)/T \end{pmatrix}$$

- **To obtain temperature dependence, certain extra assumptions are essential.**

Assumptions

- **Critical theory <u>is time reversal invariant</u>.**
  $\Rightarrow$ Hall conductivities $\propto$ B.

- **Critical theory <u>not particle-hole symmetric</u>.**
  (else thermopower, Hall conductivities, vanish, i.e. sensitive to irrelevant operators).

  (cf. Bose Hubbard model at integer filling).

Assumptions

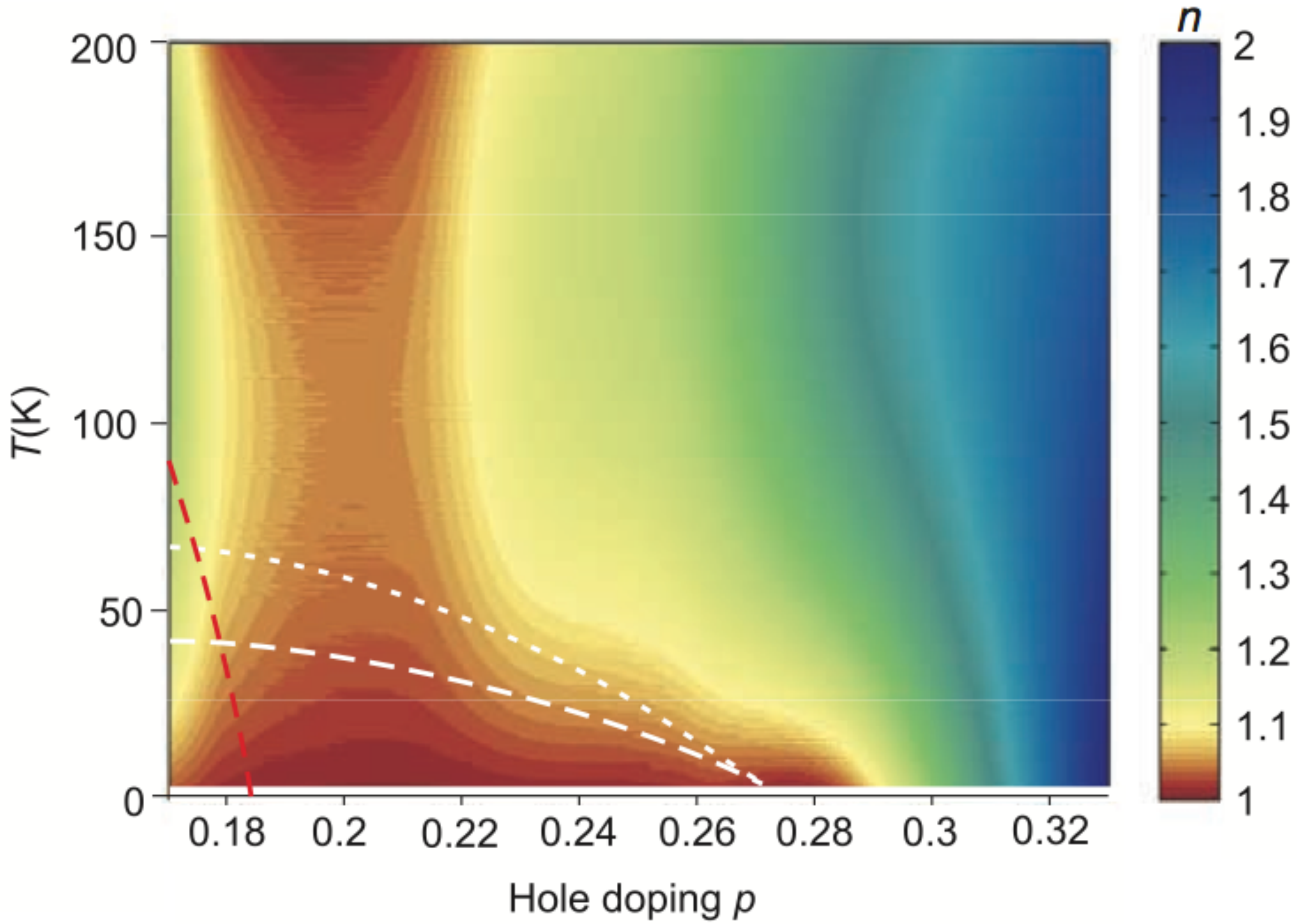
- **The electric and thermal <u>currents do not overlap with any long-lived modes</u>. In particular, momentum must degrade quickly.**

  (else conductivities are sensitive to the decay rate of these modes).

i. Linear Resistivity

$$\rho \equiv \frac{1}{\sigma_{xx}}$$

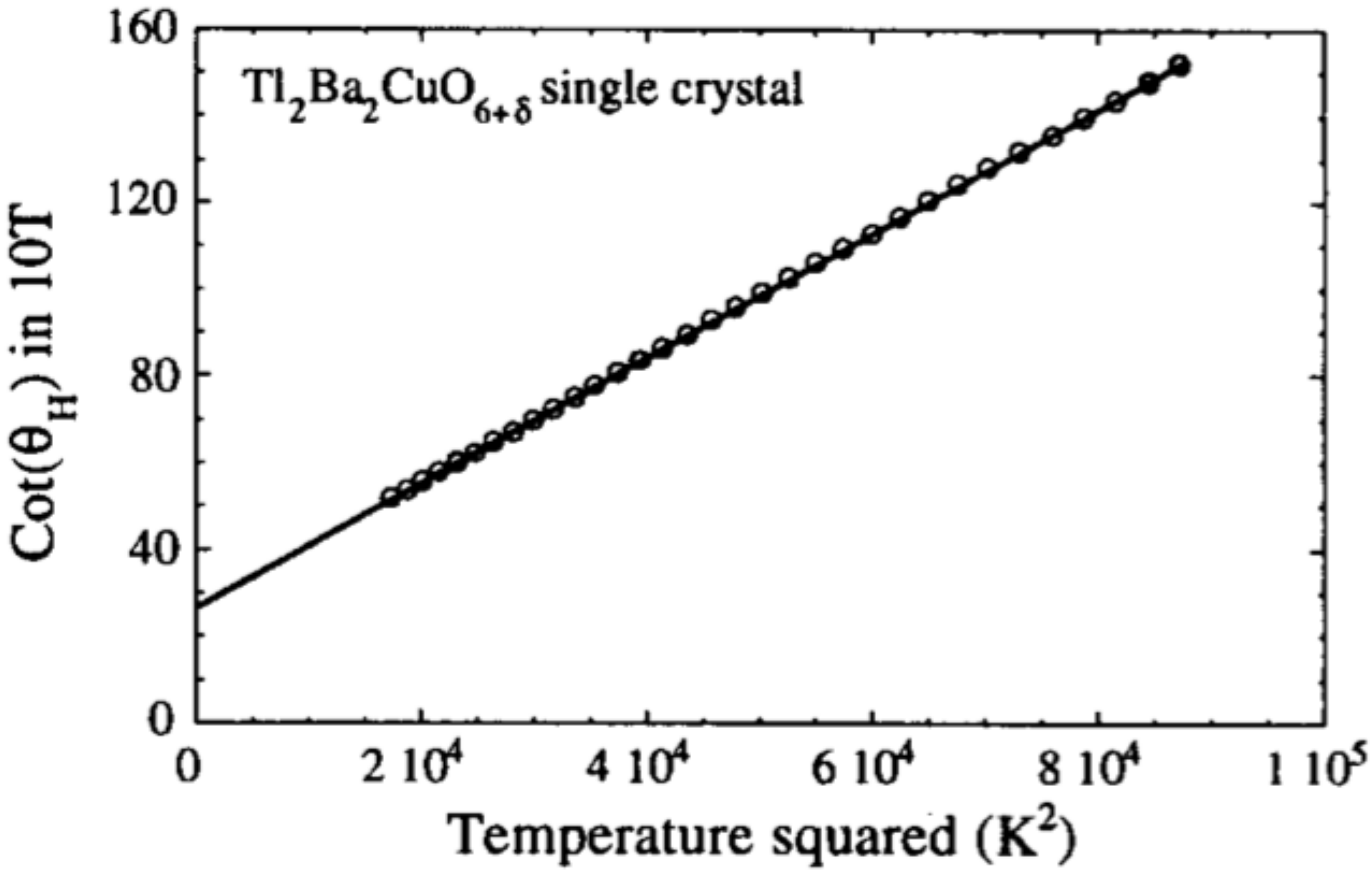
- **Observed in all cuprates.**
- **Centered at slight overdoping**



- **LSCO, [Cooper et al. Science '09]**

ii. Quadratic Hall angle

$$\cot\theta_H \equiv \frac{\sigma_{xx}}{\sigma_{xy}}$$

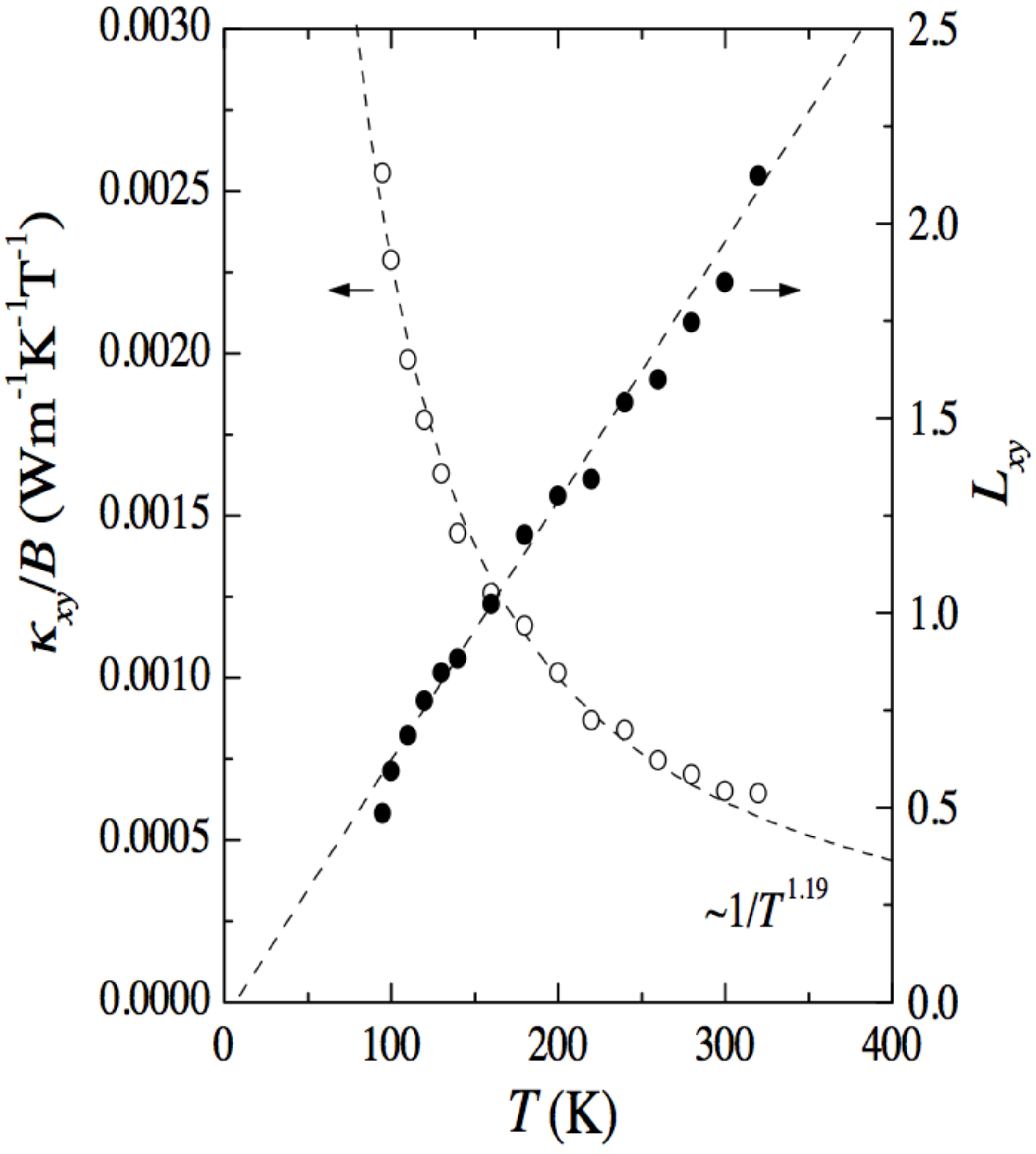


- [Tyler et al. Physica C. '97]
- **Originally found in YBCO by Chien et al. PRL '91.**

iii. Hall Lorenz ratio

$$L_H \equiv \frac{\kappa_{xy}}{T\sigma_{xy}}$$

- **Not sensitive to phonons!**
- **OP YBCO [Zhang et al. PRL '00]**



(Caveat, recently Matusiak et al. have reported results differing by an order of magnitude!)

iii. Hall Lorenz ratio

- **Anything other than constant requires a nonzero: $-2\Phi = z$!**
- **Directly measures the relative importance of charge versus heat.**

Exponents

- The three measurements above fix:

$$z = \frac{4}{3}\,, \qquad \theta = 0\,, \qquad \Phi = -\frac{2}{3}\,.$$

- From these we can make some predictions.

- The exponents imply that B/T^2 is dimensionless. All observables should be scaling functions of this quantity in the strange metal.

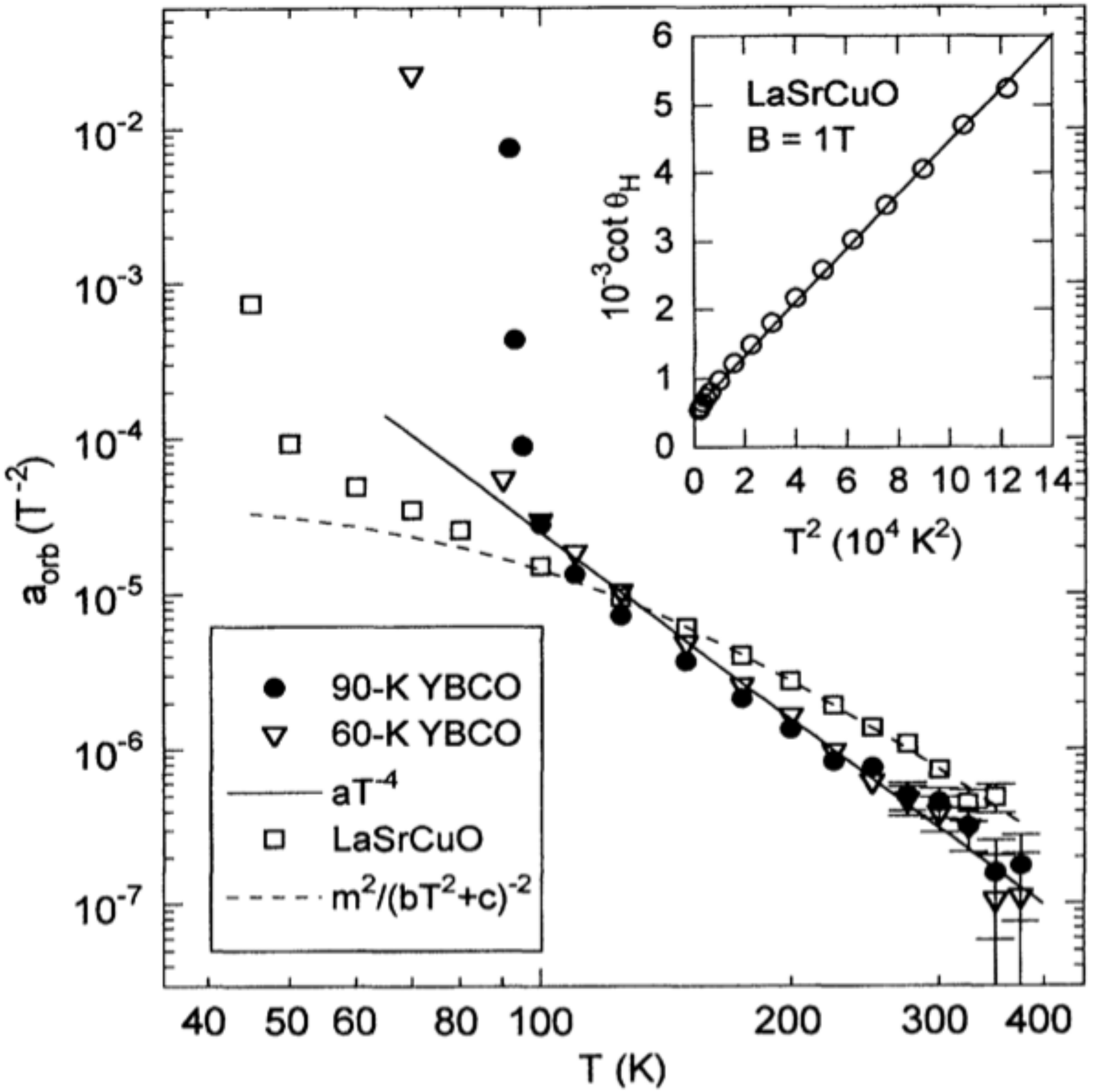
iv. Magnetoresistance

$$\frac{\Delta\rho}{\rho} \equiv \frac{\rho_{xx}(B) - \rho_{xx}(0)}{\rho_{xx}(0)}$$

- Our prediction is

$$\frac{\Delta\rho}{\rho} \sim \frac{B^2}{T^4}$$

- Agrees!
- Also observed in Tl2201.



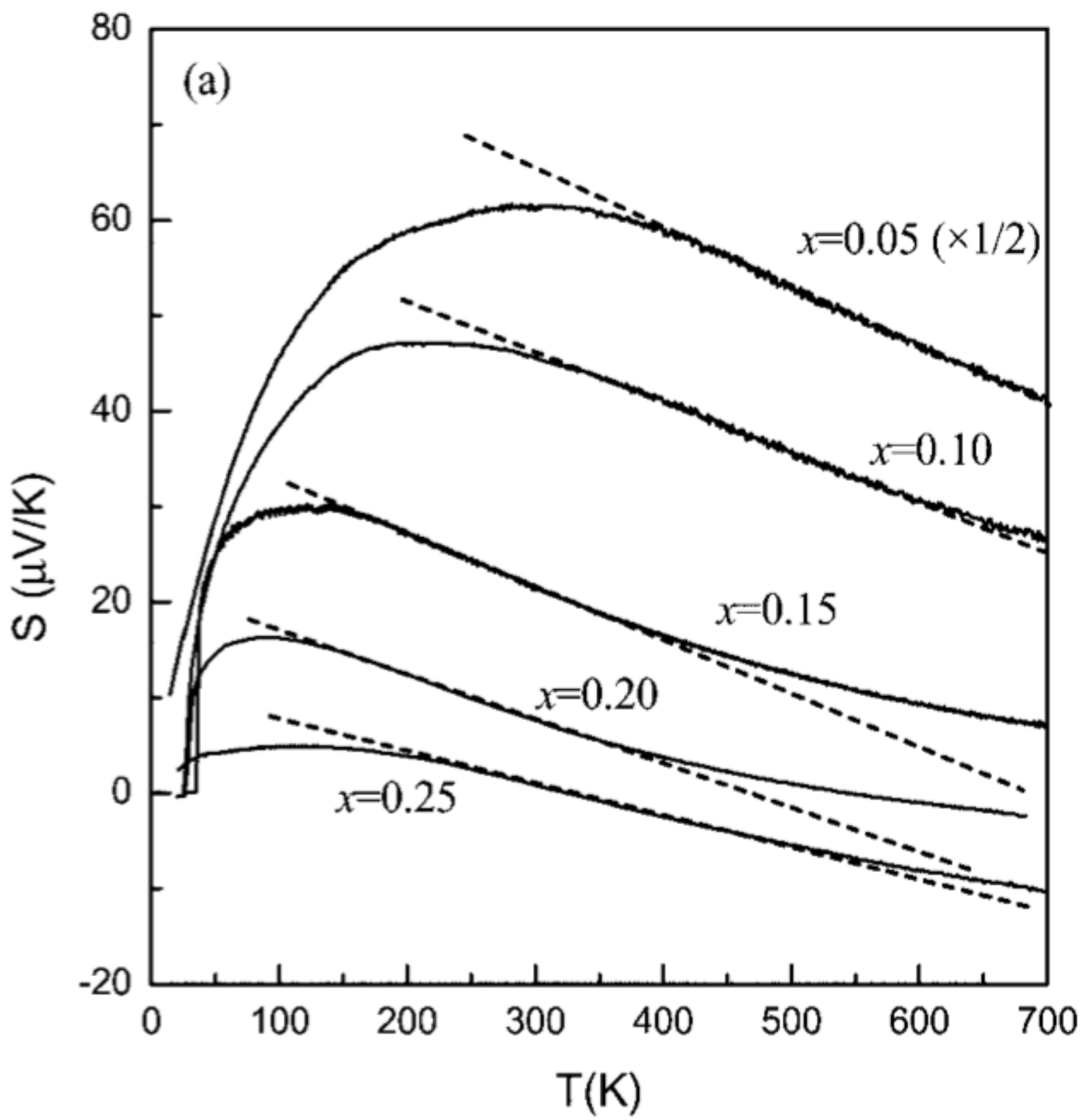
v. Thermopower

$$S \equiv \frac{\alpha_{xx}}{\sigma_{xx}}$$

- Our prediction is

$$S \sim -T^{1/2}$$

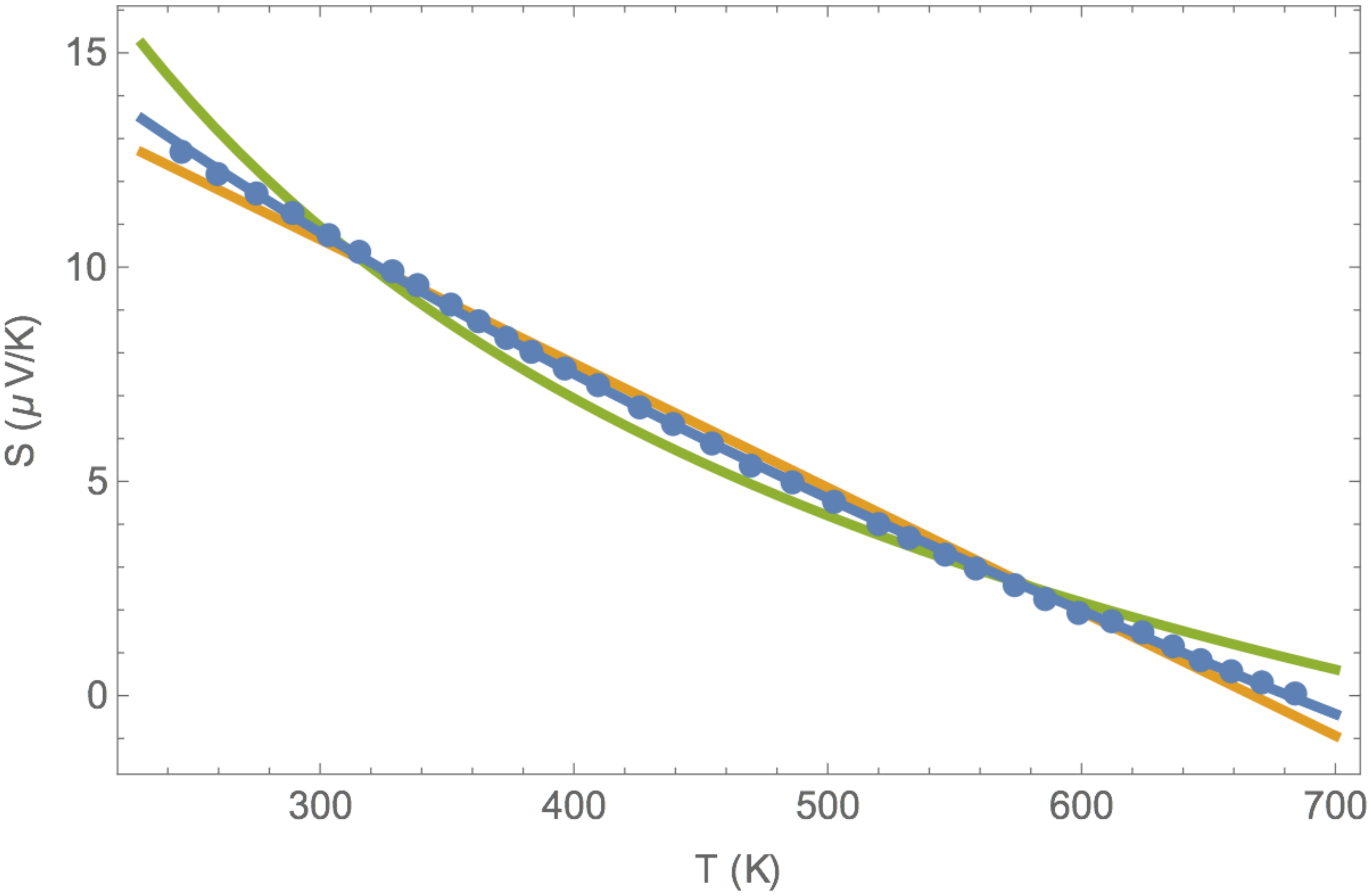
- Need high quality data over wide temperature range at slight overdoping.



[Kim et al. Ann. Phys. '04]

v. Thermopower

- **Data at overdoping and high enough temperature fits well to:** $S \sim a - bT^{1/2}$**!**



Predictions

- **Nernst:**

$$\nu \equiv \frac{1}{B}\left(\frac{\alpha_{xy}}{\alpha_{xx}} - S\tan\theta_H\right) \sim T^{-3/2}$$

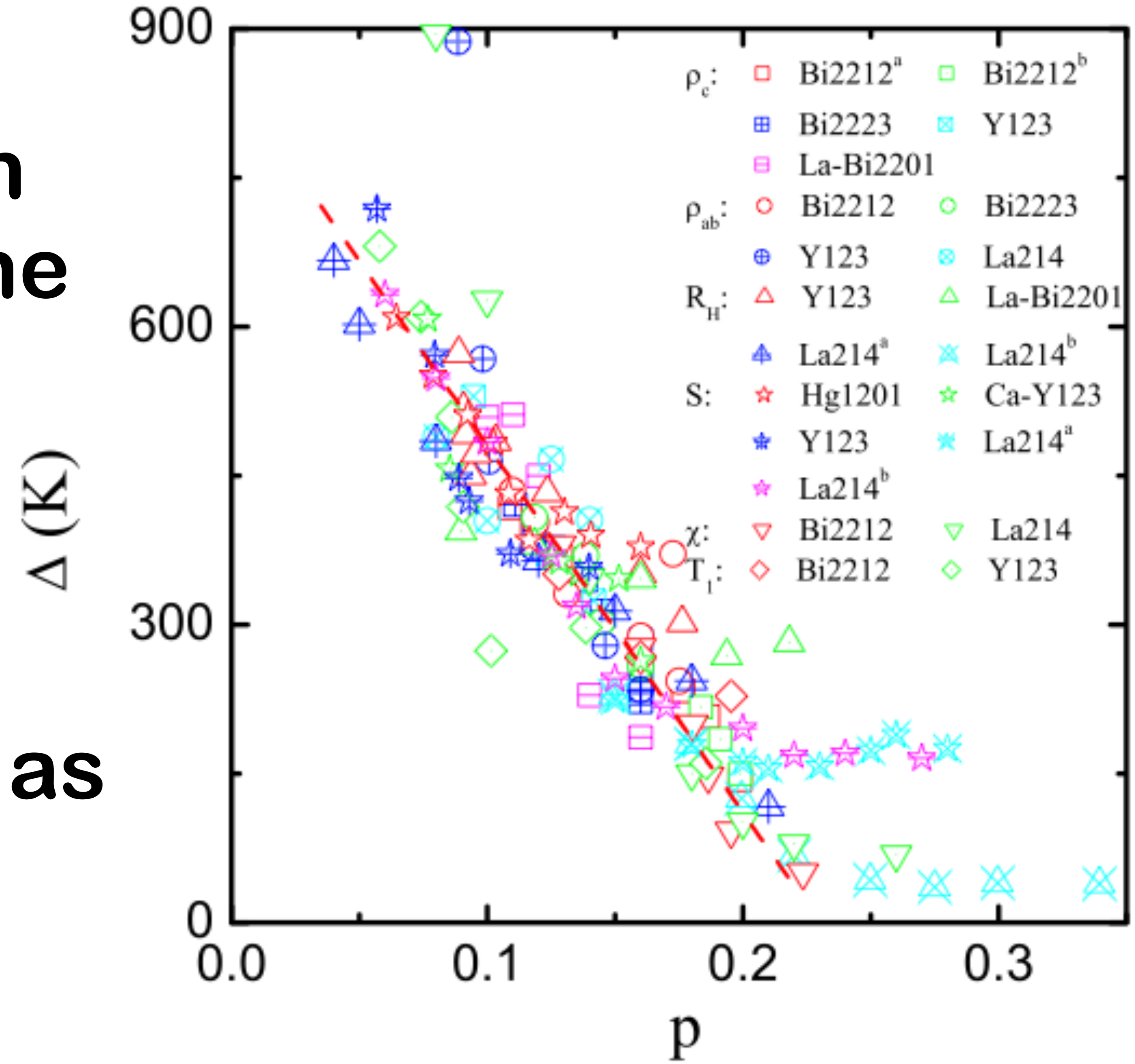
- **Thermal conductivity:**

$$\kappa_{xx} \sim T$$

- **The scaling regime may not extend to T = 0. The best regions seems to be slightly overdoped.**

The pseudogap

- According to our scaling, n and T have the same units.
- Predicts pseudogap should close as $T^* \sim (p-p_c)$.

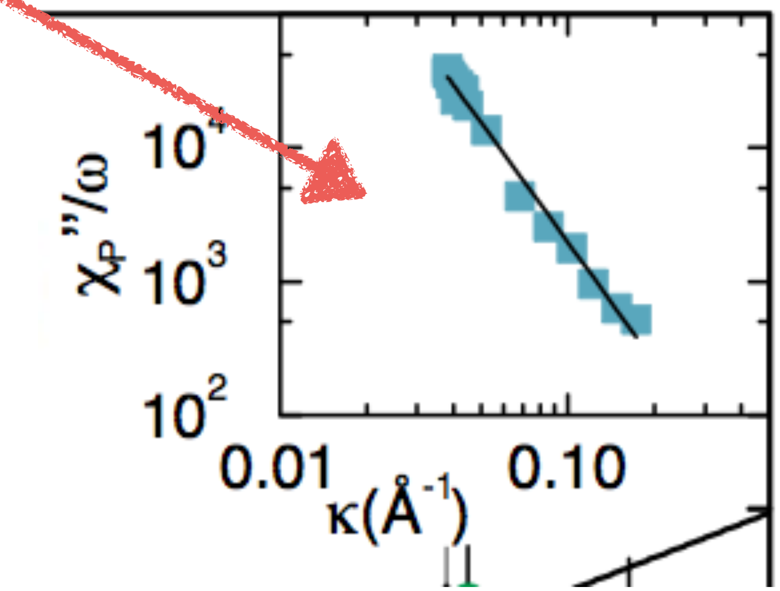


[Luo et al. PRB '08]

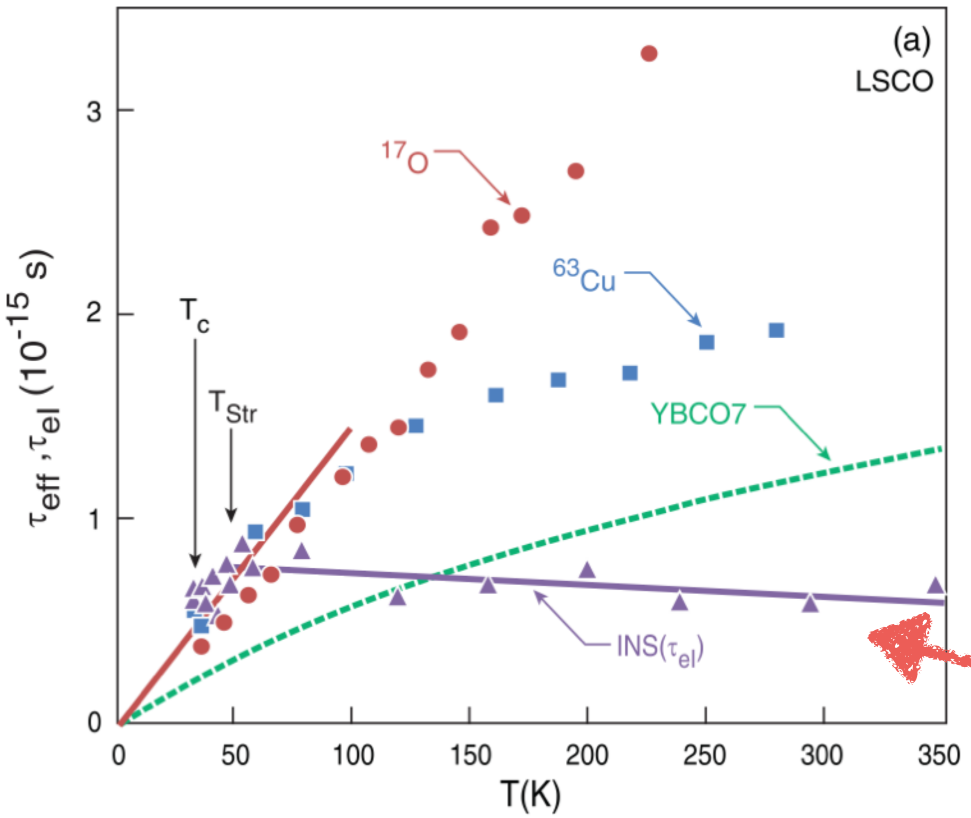
Spin susceptibility

- **Scaling peak observed in spin susceptibility in underdoped LSCO:**

$$\lim_{\omega\to 0} \frac{\chi''(\omega, q_\star, T)}{\omega} \sim \frac{1}{[\Delta_q(T)]^{3\pm 0.3}}$$



[Aeppli '97]



$$\tau_{\text{eff}} \equiv T \int d^2q \lim_{\omega\to 0} \frac{\chi''(\omega, q, T)}{\omega} \sim \text{const.},$$

Spin susceptibility

- **If we assume magnetic field couples to spin with same scaling as to current:**

$$\lim_{\omega\to 0}\frac{\chi''(\omega,q_\star,T)}{\omega}\sim[\Delta_q(T)]^{-\theta+2\Phi-2}\sim\frac{1}{[\Delta_q(T)]^{10/3}}$$

$$\tau_{\rm eff}\sim T^{(-\theta+2\Phi+z)/z}\sim {\rm const.}\,,$$

- **Works! But possibly a coincidence.**

Thermodynamics

- The strange metal thermodynamics is surprisingly conventional. Observationally:

$$c \sim T$$
$$\chi \sim \text{const.}$$

- While our scaling predicts:

$$c \sim T^{3/2}$$
$$\chi \sim T^{-3/2}$$

- Non-critical background dominates? Spinon Fermi surface? Hot spots? Localized d.o.f.?

Summary

- Electrical and heat currents remain well defined operators in absence of quasiparticle description.
- Attempted to match data using three exponents.
- Works well for transport, anomalous dimension for charge density crucial!
- Predictions such as B/T^2 scaling.

To do

- If indeed the strange metal is described by an incoherent scaling theory, should describe onset of superconductivity from this context.

- Natural quantity is the scaling dimension of the Cooper pair operator. Does this control any known physics?