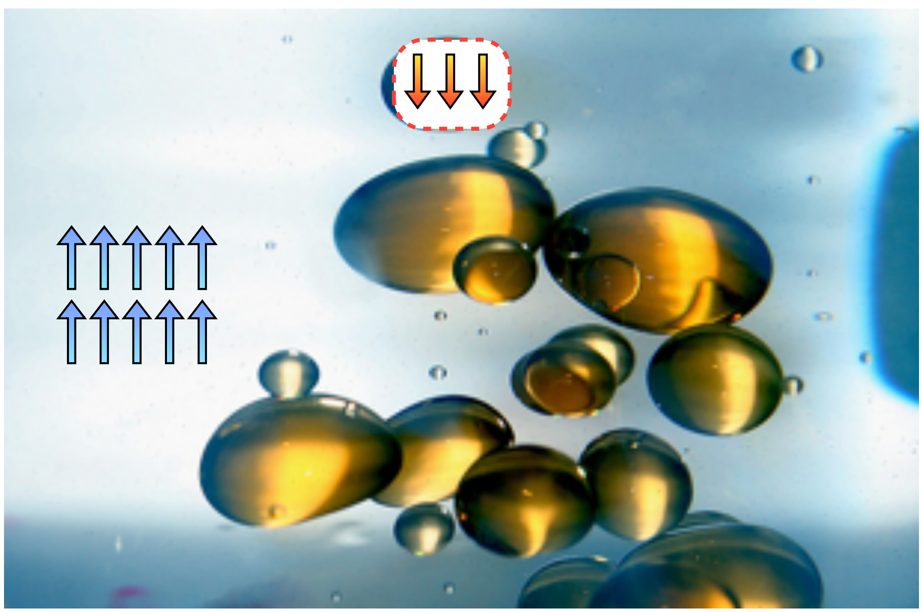
Quantum Lifshitz point and a multipolar cascade for frustrated ferromagnets

Leon Balents, KITP, UCSB



Progress and Applications of Modern Quantum Field Theory, Aspen, Feb. 2015

Collaborators

Oleg Starykh
U. Utah

Teddy Parker
UCSB

What this talk is not

- (almost) nothing topological
- No gauge fields
- Nothing fractional
- No anyons. Not even fermions
- No CFT, no bootstrap

What this talk is not

- (almost) nothing topological
- No gauge fields
- Nothing fractional
- No anyons. Not even fermions
- No CFT, no bootstrap
- Not even a complete solution 

What it is about

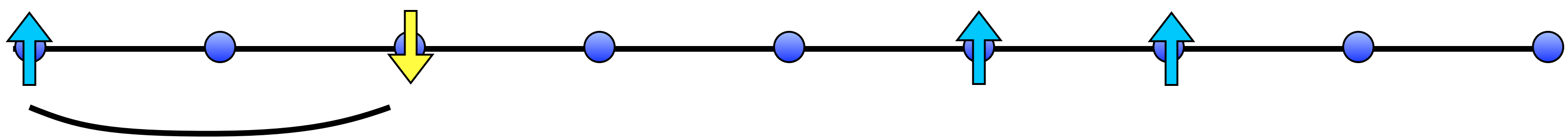
- I will discuss the simplest example of a "frustrated ferromagnet", and argue that there is a simple QFT description of such systems, with surprisingly rich phenomenology
- It is clear that this description extends to higher dimensions and perhaps the phenomenology does as well

Outline

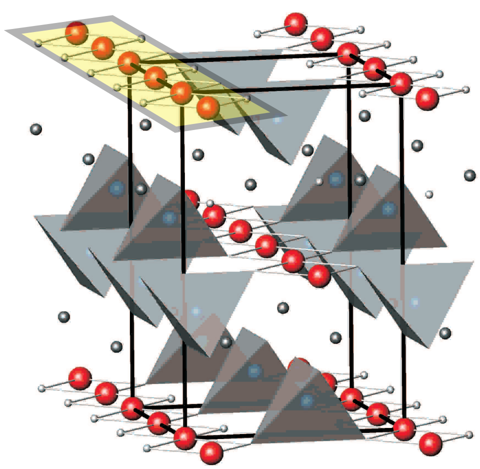
- Introduction and phenomena:
  - a QCP, and multipolar phases
- QFT: what we need
- Lifshitz NLsM
  - Limits and analysis

Frustrated ferromagnet

1d S=1/2 chain



$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$



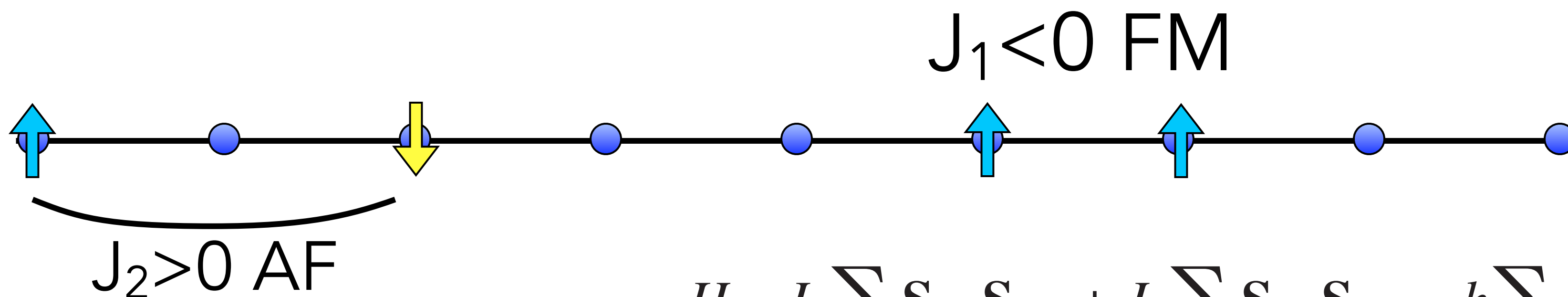
$LiCuVO_4$

Compound	J_1, J_2 (K)	∠ Cu-O-Cu (deg)	T_N (K)	H_s (T)
Li_2ZrCuO_4[12, 13]	−151, 35	94.1	6.4	-
$Rb_2Cu_2Mo_3O_{12}$[14, 15]	−138, 51	89.9, 101.8	< 2	14
		91.9, 101.1		
$PbCuSO_4(OH)_2$[16–18]	−100, 36	91.2, 94.3	2.8	5.4
$LiCuSbO_4$[19]	−75, 34	89.8, 95.0	< 0.1	12
		92.0, 96.8		
$LiCu_2O_2$[20–22]	−69, 43	92.2, 92.5	22.3	110
$LiCuVO_4$[23–31]	−19, 44	95.0	2.1	44.4
$NaCuMoO_4(OH)$	−51, 36	92.0, 103.6	0.59	26

K. Nawa *et al*, arXiv:1409.1310

Frustrated ferromagnet

1d S=1/2 chain



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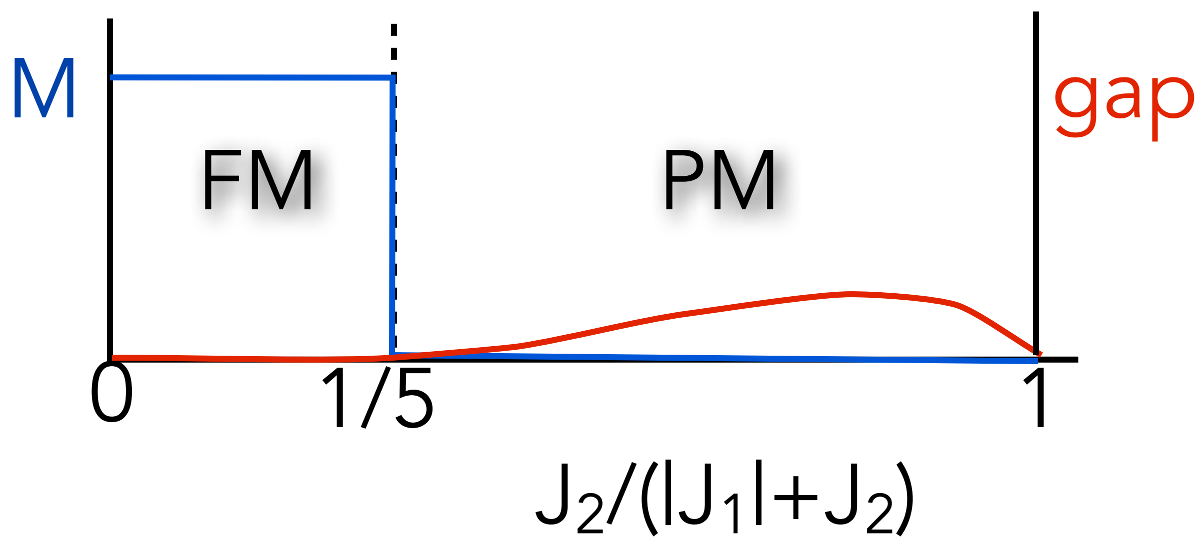
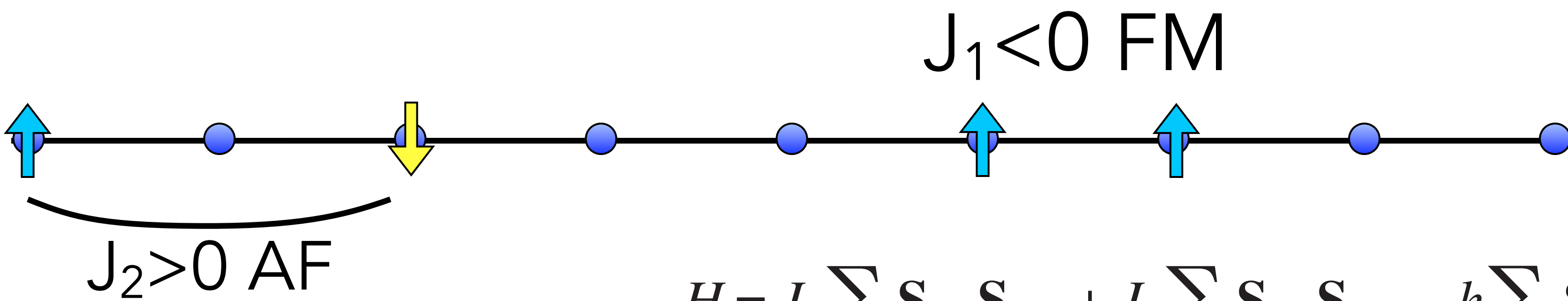


Frustrated ferromagnet

1d S=1/2 chain



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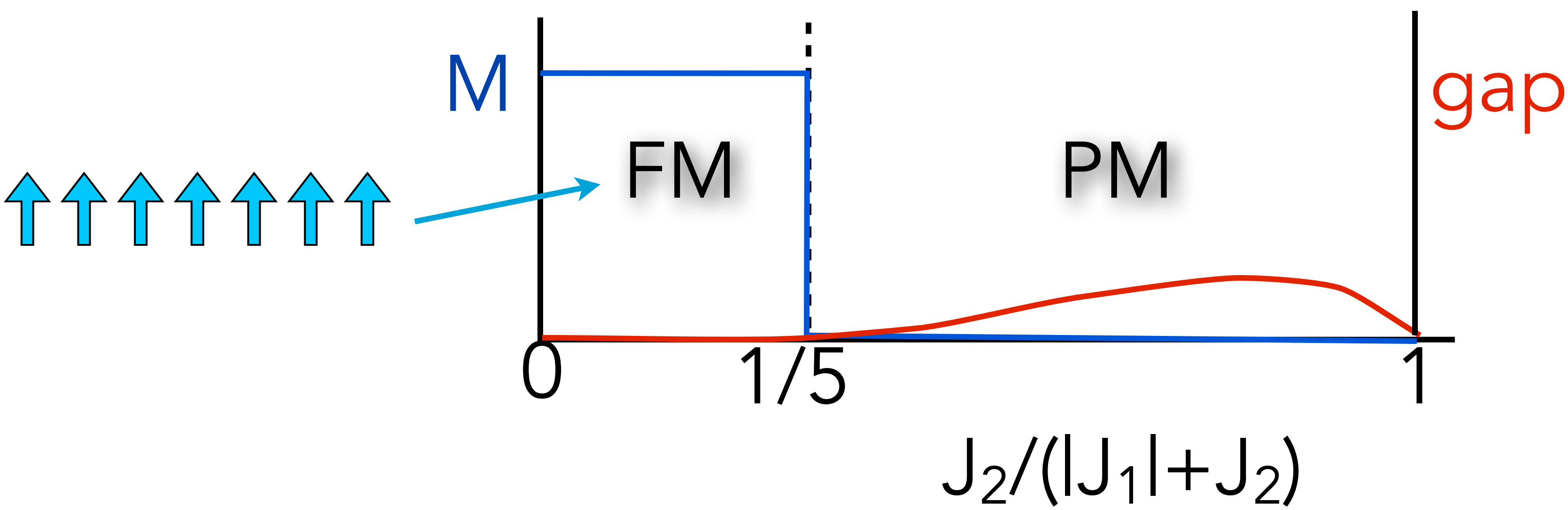
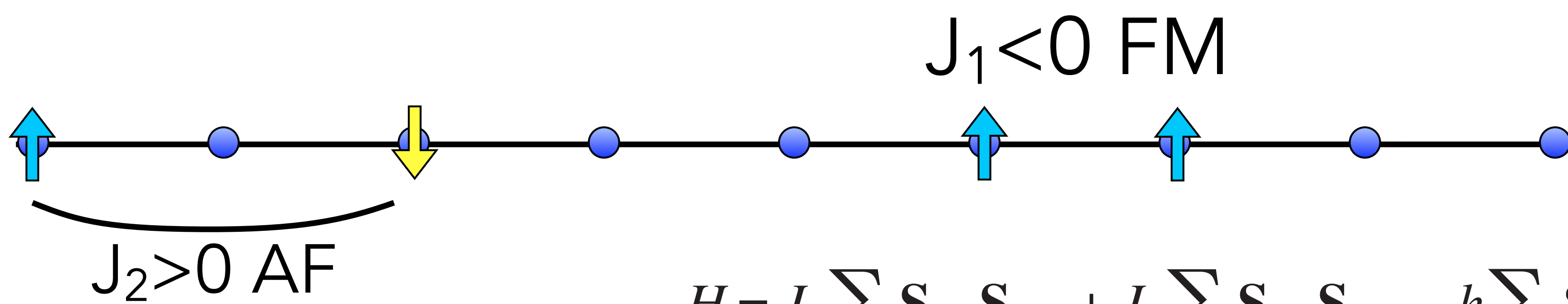


Frustrated ferromagnet

1d S=1/2 chain



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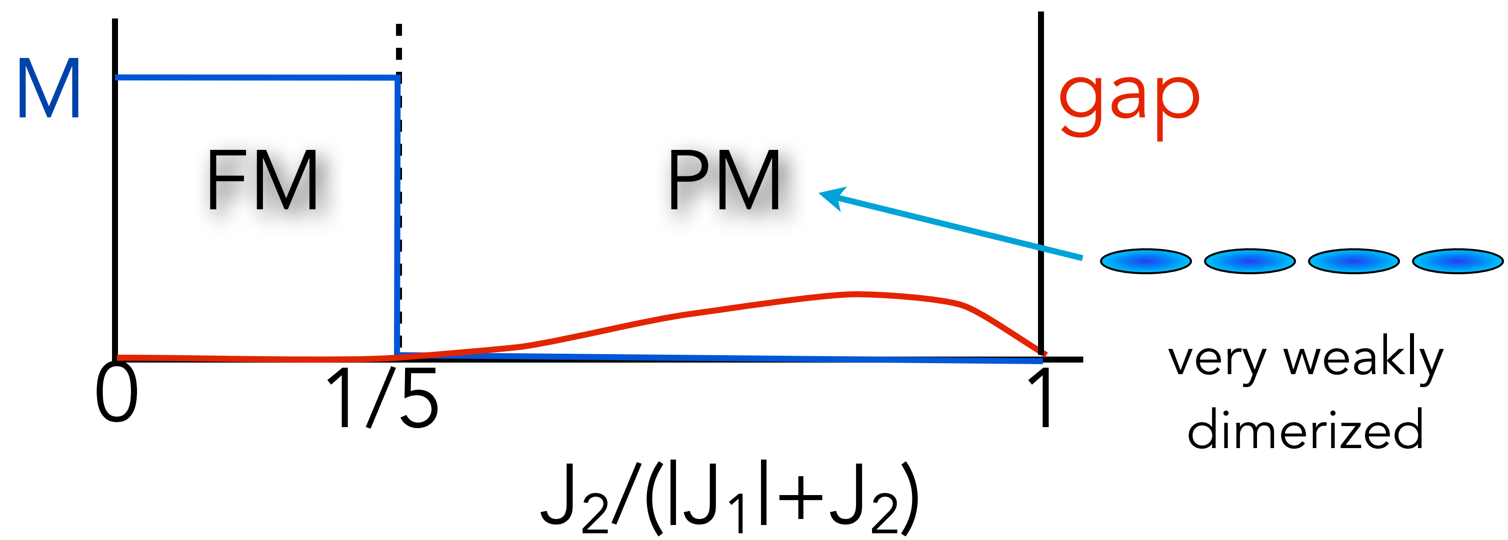
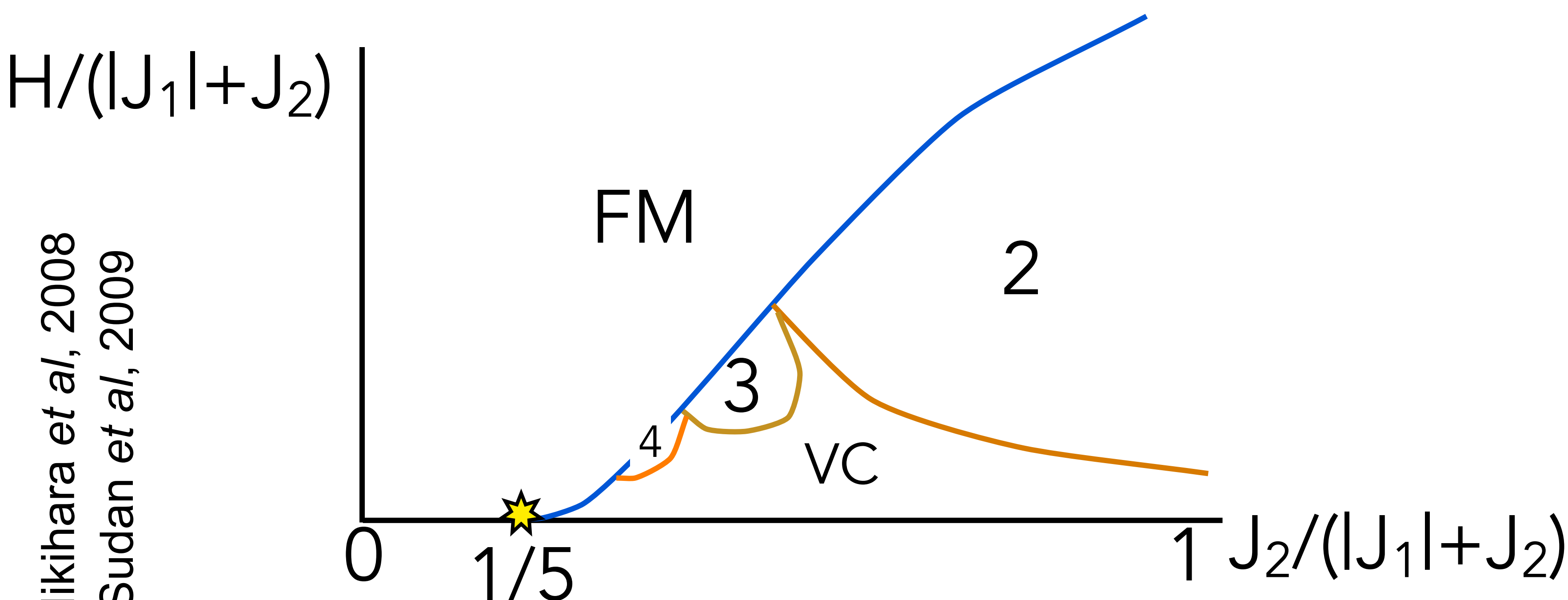
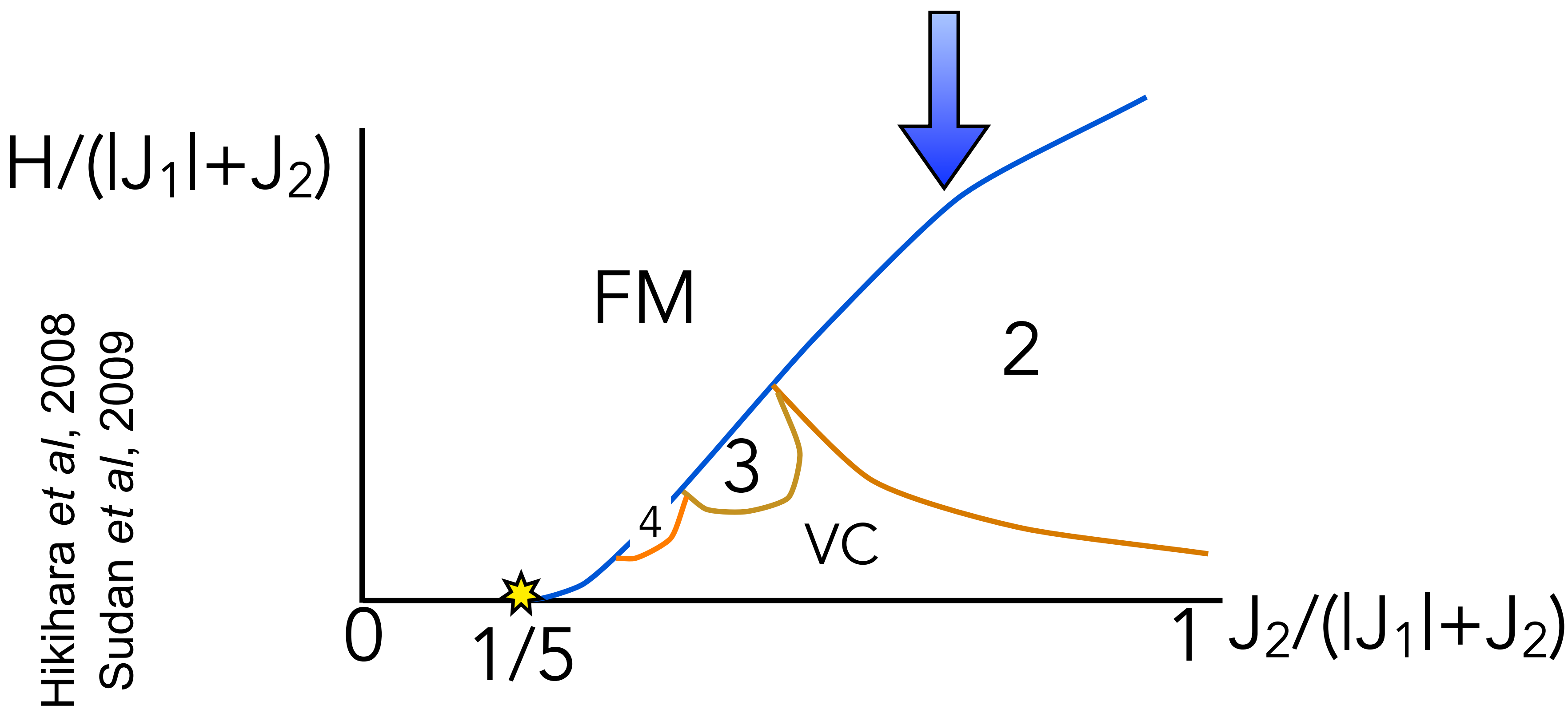


Multipolar phases



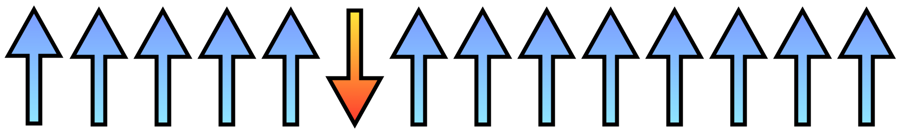
Hikihara *et al*, 2008
Sudan *et al*, 2009

Multipolar phases

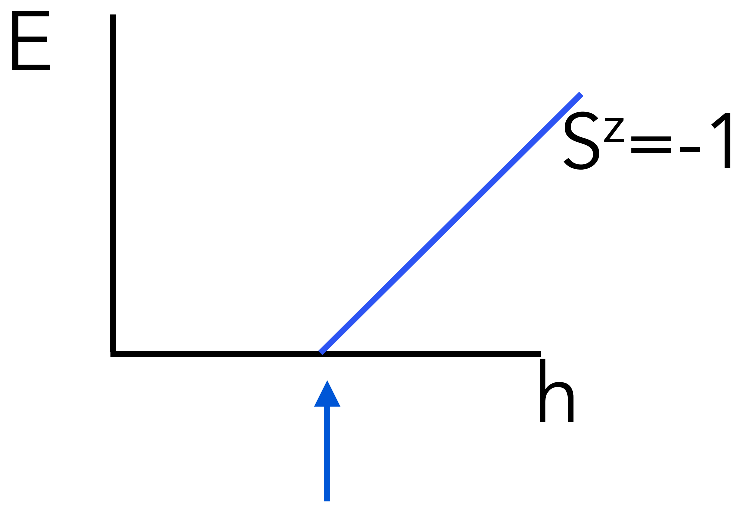


Magnon BEC

1-magnon



$E - E_{FM} = \varepsilon_1 + h$

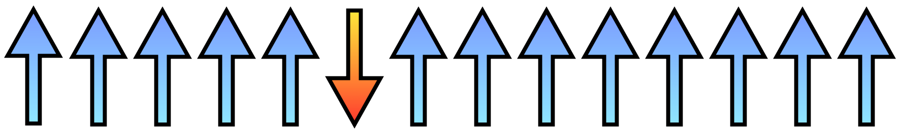


Magnon (quasi-)BEC

$$\langle S_i^- \rangle \sim \Psi e^{iqx_i}$$

Magnon BEC

1-magnon



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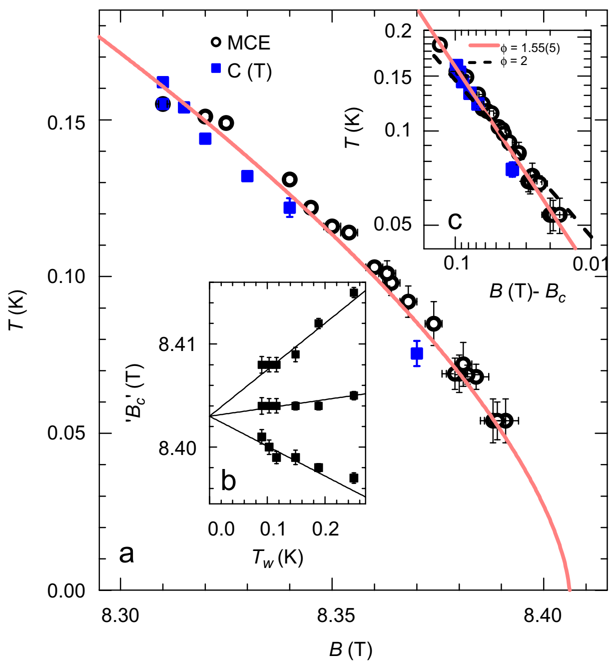
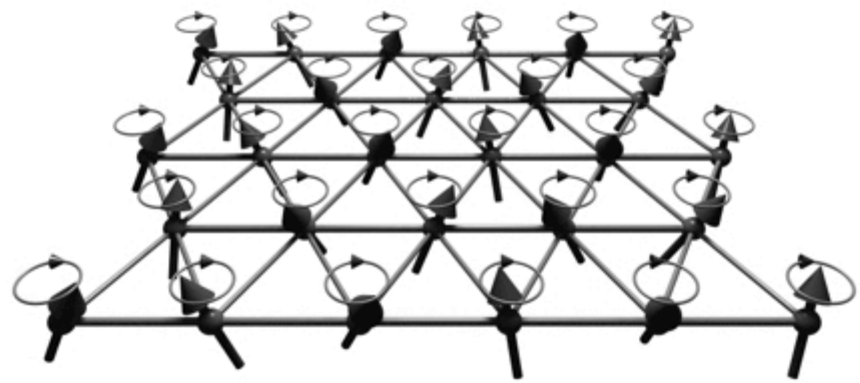
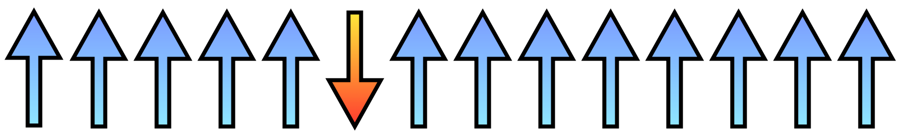


T. Radu *et al*, 2007

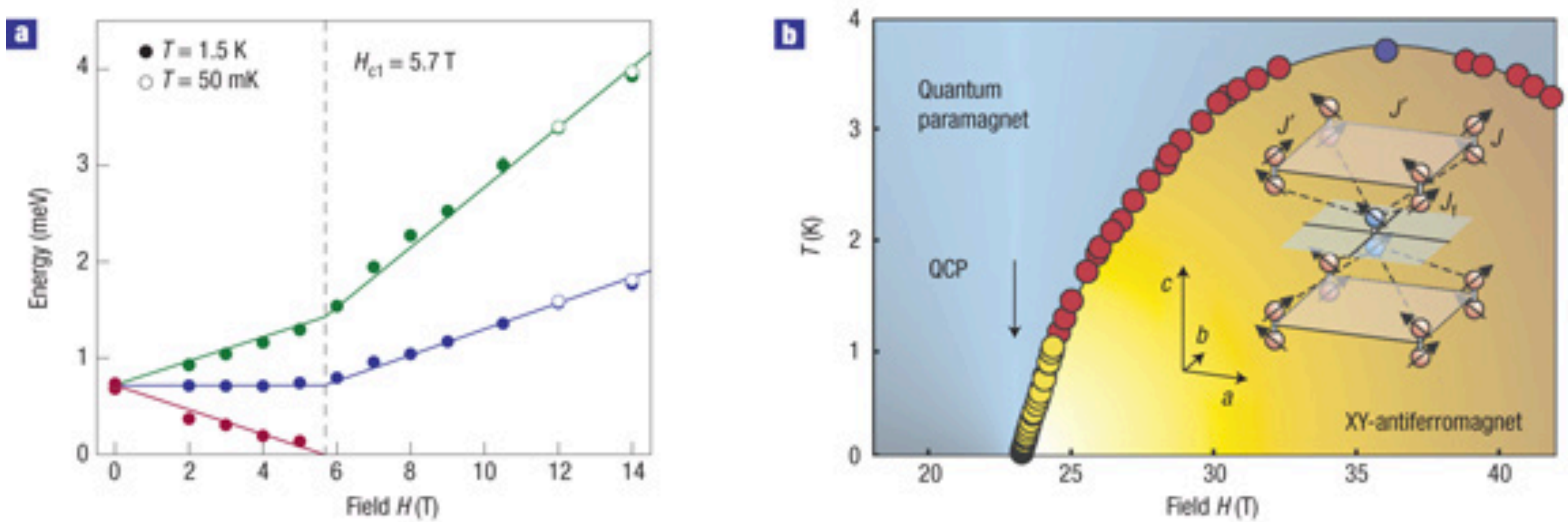


Magnon BEC

1-magnon



$E\text{-}E_{FM} = \varepsilon_1 + h$

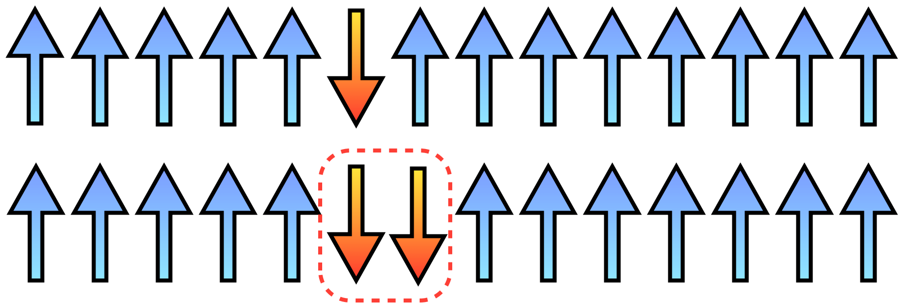


T. Giamarchi *et al*, 2008

Magnon binding

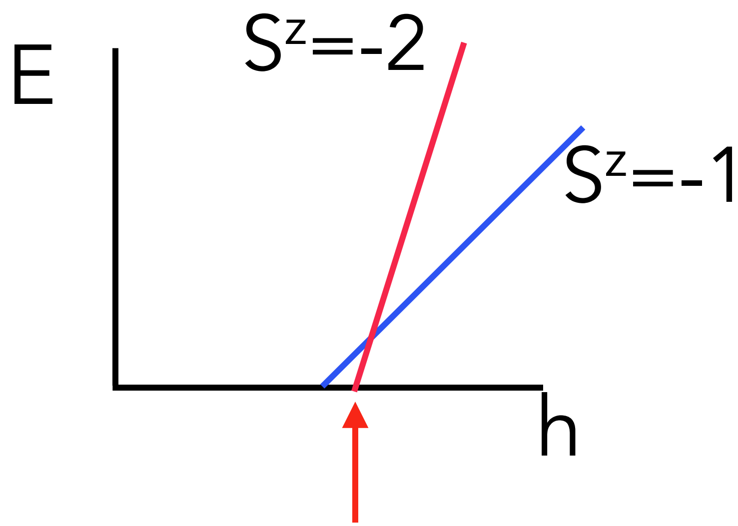
1-magnon

2-magnon bound state



$E\text{-}E_{FM} = \varepsilon_1 + h$

$E\text{-}E_{FM} = \varepsilon_2 + 2h$



$\varepsilon_2 < 2\,\varepsilon_1$: "molecular" bound state

Formation of molecular fluid

For d>1 at T=0 this is a molecular BEC = true spin nematic

Hidden order

No dipolar order

$$\langle S_i^z \rangle - M = 0 \quad \langle S_i^+ S_j^- \rangle \sim e^{-|i-j|/\xi}$$

$$\langle S_i^{\pm} \rangle = 0$$

S^z=1 gap

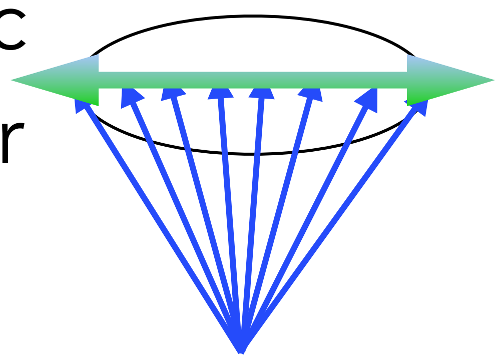
Nematic order

$$\langle S_i^+ S_{i+a}^+ \rangle \neq 0$$

Magnetic quadrupole moment

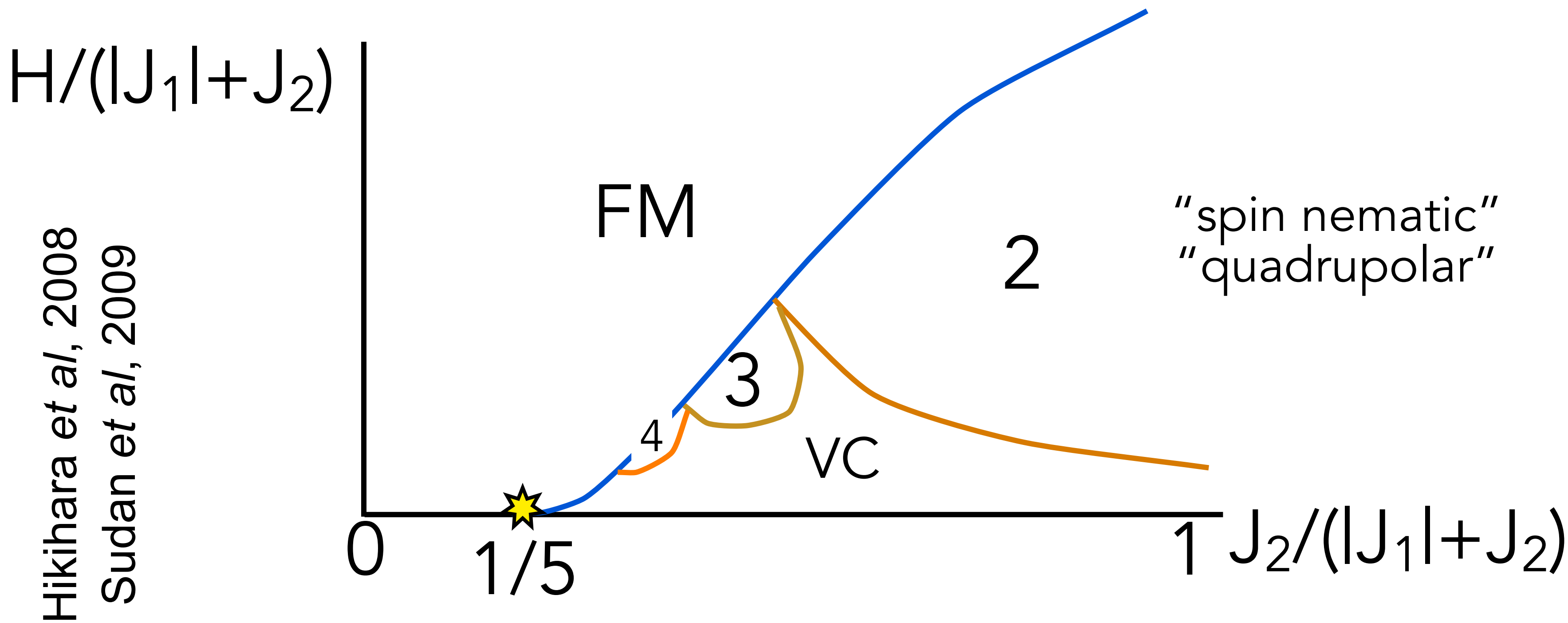
Symmetry breaking U(1) → Z_2

nematic director



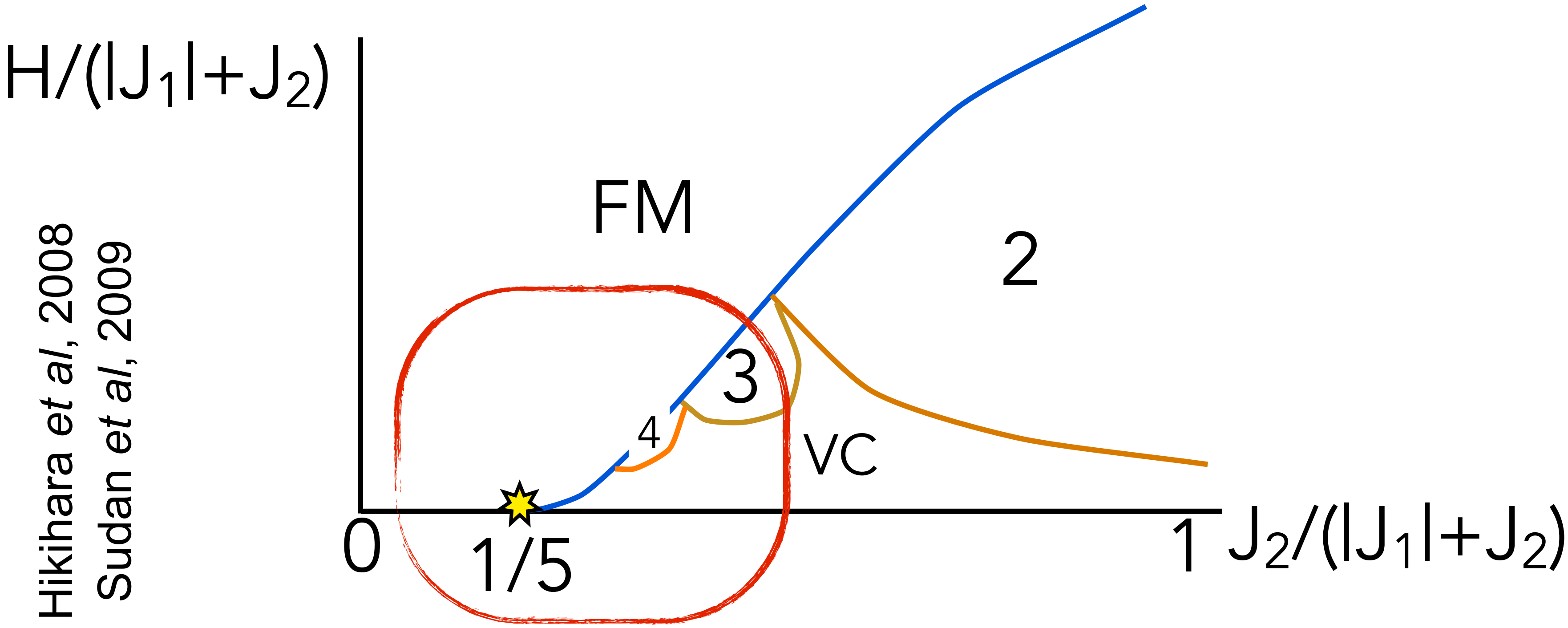
can think of a fluctuating fan state

Multipolar phases



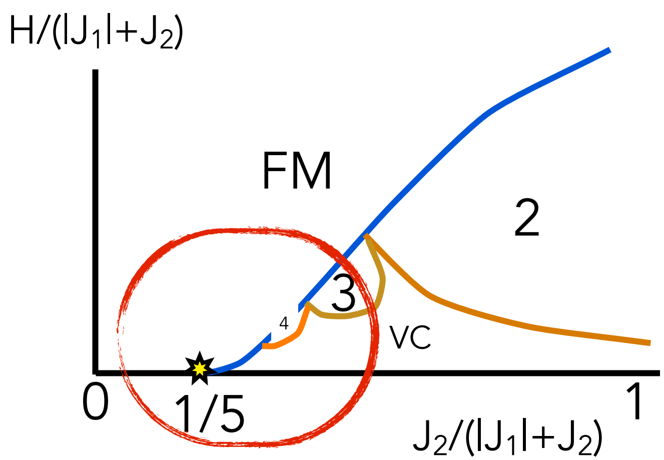
A progression of higher and higher multipolar phases on approaching the QCP!

Multipolar phases



Is there a QFT that describes this region?

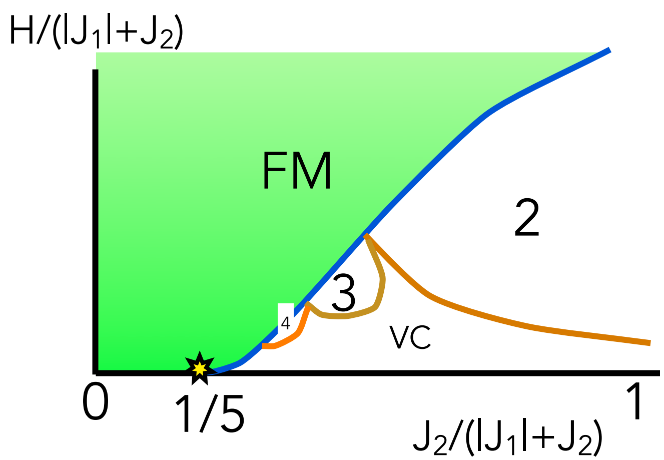
A QFT?



$$\Psi_n \sim \langle \left(S^-\right)^n \rangle$$

- Is this behavior generic?
- Is the cascade infinite, or does it terminate?
- Can a single QFT describe an infinite number of order parameters?
- Is this specific to one dimension?

A QFT?



$$\Psi_n \sim \langle \left(S^-\right)^n \rangle$$

- A strong constraint:
  - Entire green area *including the QCP itself* has *exact* trivial FM ground state
  - Not a CFT

Lifshitz Point

- Effective action - NLσM $|\hat{m}| = 1$

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

WZW/Berry phase term

tunes QCP

two symmetry allowed interactions at $O(q^4)$

$$\mathcal{A}_B = \frac{\hat{m}_1 \partial_\tau \hat{m}_2 - \hat{m}_2 \partial_\tau \hat{m}_1}{1 + \hat{m}_3}.$$

All properties near Lifshitz point obey "one parameter universality" dependent upon u/K ratio

Lifshitz Point

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

- Intuition: behavior near the Lifshitz point should be semi-classical, since "close" to FM state which is classical

$$x \to \sqrt{\frac{K}{|\delta|}} x \qquad \tau \to \frac{K}{\delta^2} \tau$$

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \text{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

Large parameter:
saddle point!

$$v = \frac{u}{K} \qquad \overline{h} = \frac{hK}{\delta^2}$$

Lifshitz point

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

v derives from quantum fluctuations

Need it be positive?

$$\hat{m} \cdot \hat{m} = 1 \qquad \partial_x \hat{m} \cdot \partial_x \hat{m} = -\hat{m} \cdot \partial_x^2 \hat{m} \leq |\partial_x^2 \hat{m}|$$

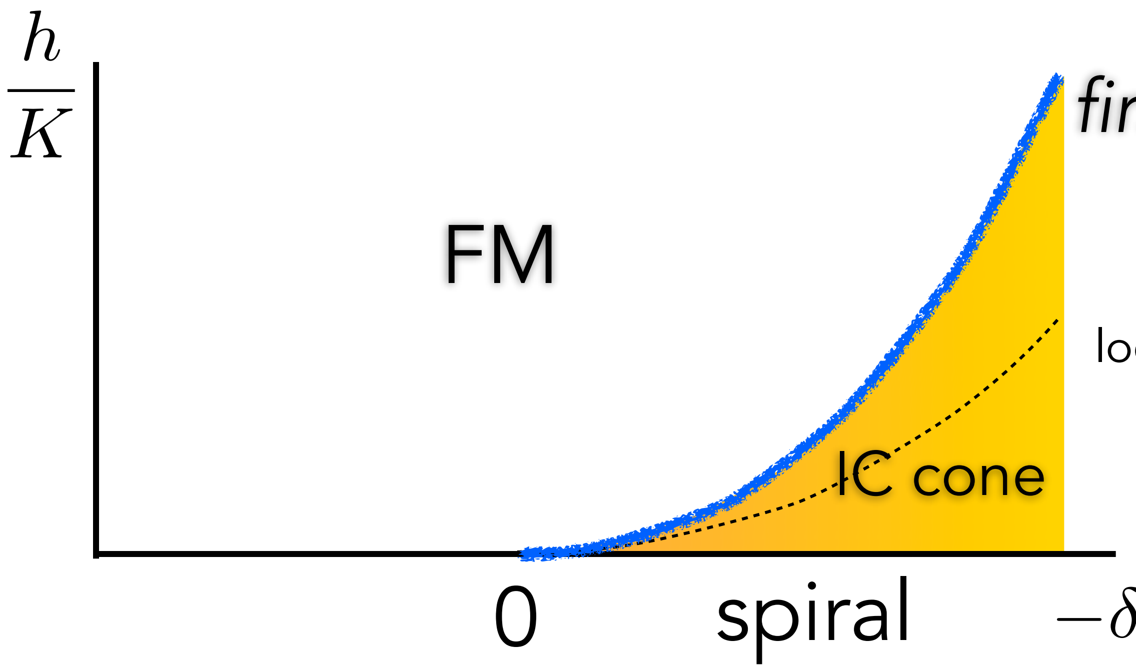
Theory is stable for v>-1

In fact, v<0

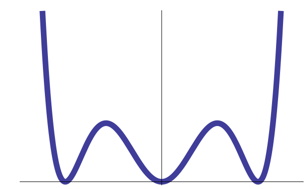
- Semiclassical large s limit: $v \sim -3/2s$
- s=1/2 exact 2-magnon calculation $v = -5/8$

Saddle point

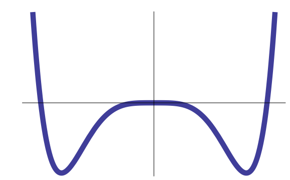
$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \text{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$



first order



local instability of FM state

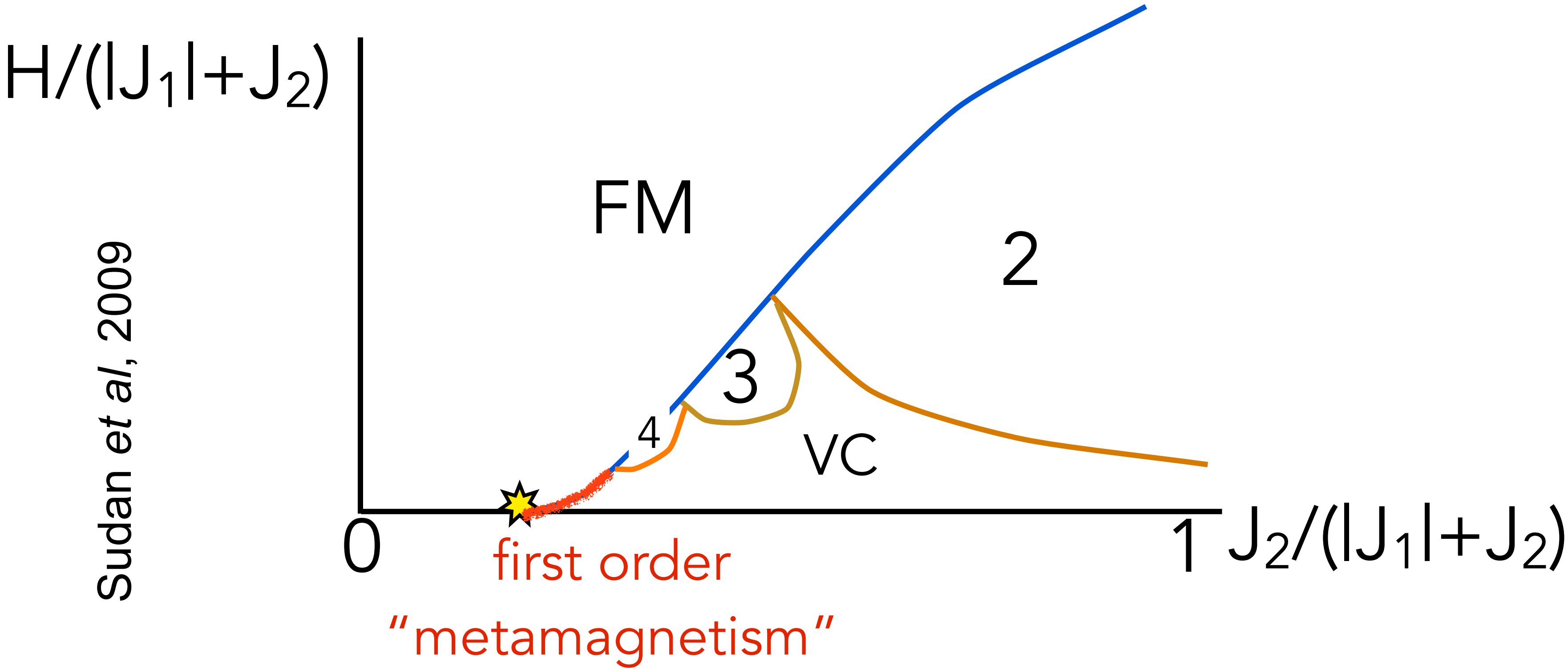


$$\hat{m} = \begin{pmatrix} |\Psi| \cos(qx + \phi) \\ \pm|\Psi| \sin(qx + \phi) \\ \sqrt{1 - |\Psi|^2} \end{pmatrix}$$

$$-1 < v < -\tfrac{1}{4}$$

$$h_c = \frac{\delta^2}{8K\sqrt{|v|}(1 - \sqrt{|v|})} > \frac{\delta^2}{2K}$$

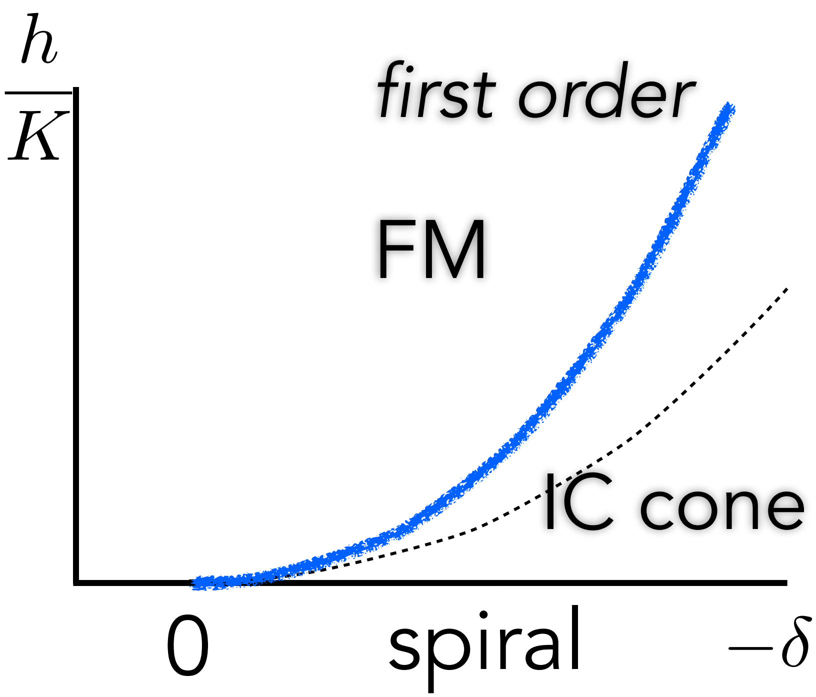
Multipolar phases



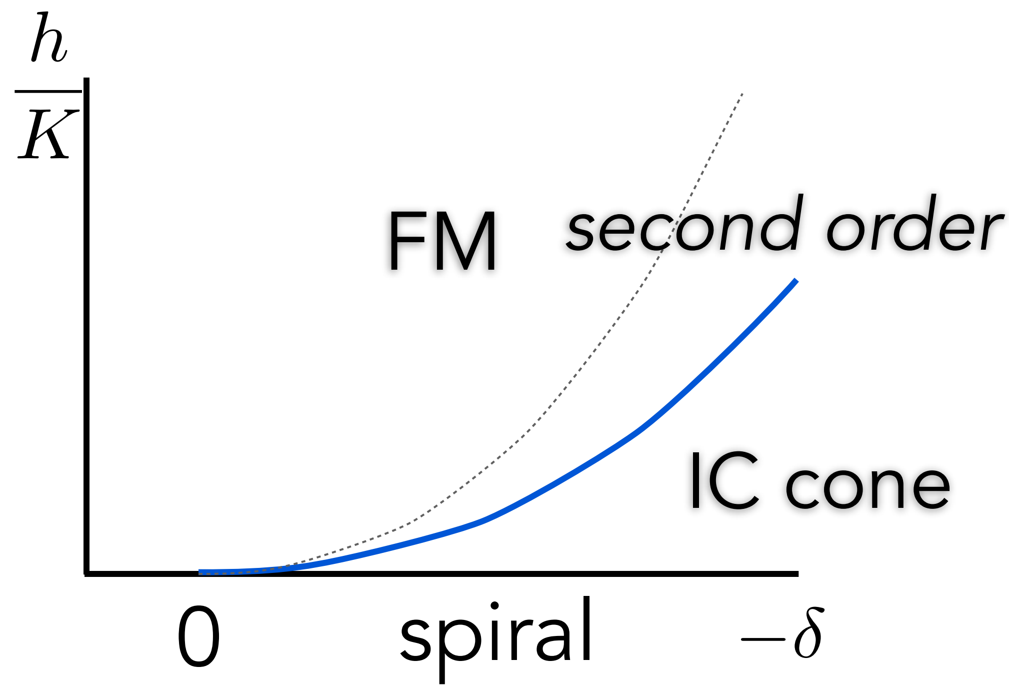
Sudan *et al*, 2009

Saddle point

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \text{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$



$-1 < v < -1/4$



$v > -1/4$

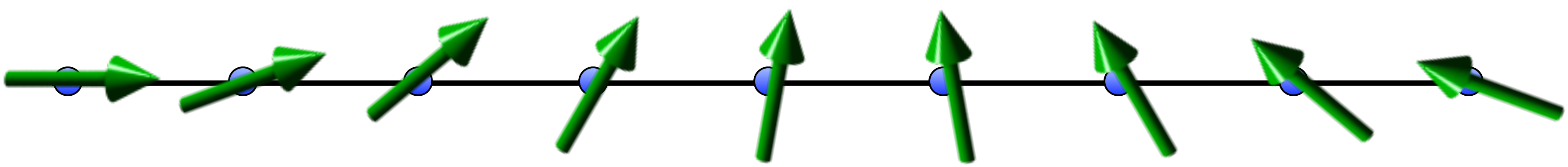
N.B.: at saddle point level there is no scale for δ

Beyond saddle point

- Issues:
  - Fate of ordered saddle points?
  - Endpoint of metamagnetic line?
  - Multipolar orders?

Zero field

- Saddle point is a spiral phase



$$\hat{m}(x) = \hat{e}_1 \cos qx + \hat{e}_2 \sin qx$$

$(\hat{e}_1, \hat{e}_2, \hat{e}_3 = \hat{e}_1 \times \hat{e}_2)$ form an SO(3) matrix

- Fluctuations are described by an SO(3) NLsM

$$S_{\text{eff}} = \frac{1}{g} \int d^2x \, \text{Tr} \, \left[(\partial_\mu O)^2 \right] + i S_{\text{topo}}$$

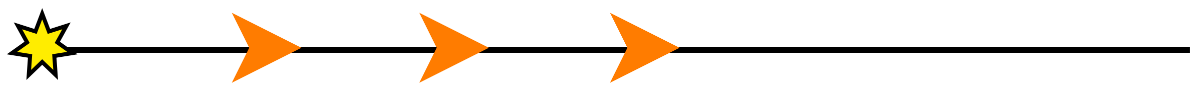
Zero field

$$S_{\text{eff}} = \frac{1}{g}\int d^2x \, \mathrm{Tr}\left[(\partial_\mu O)^2\right] + iS_{\text{topo}}$$

- NLsM is asymptotically free

gap

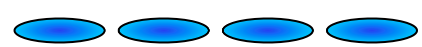
$\Delta \sim e^{-1/g}$



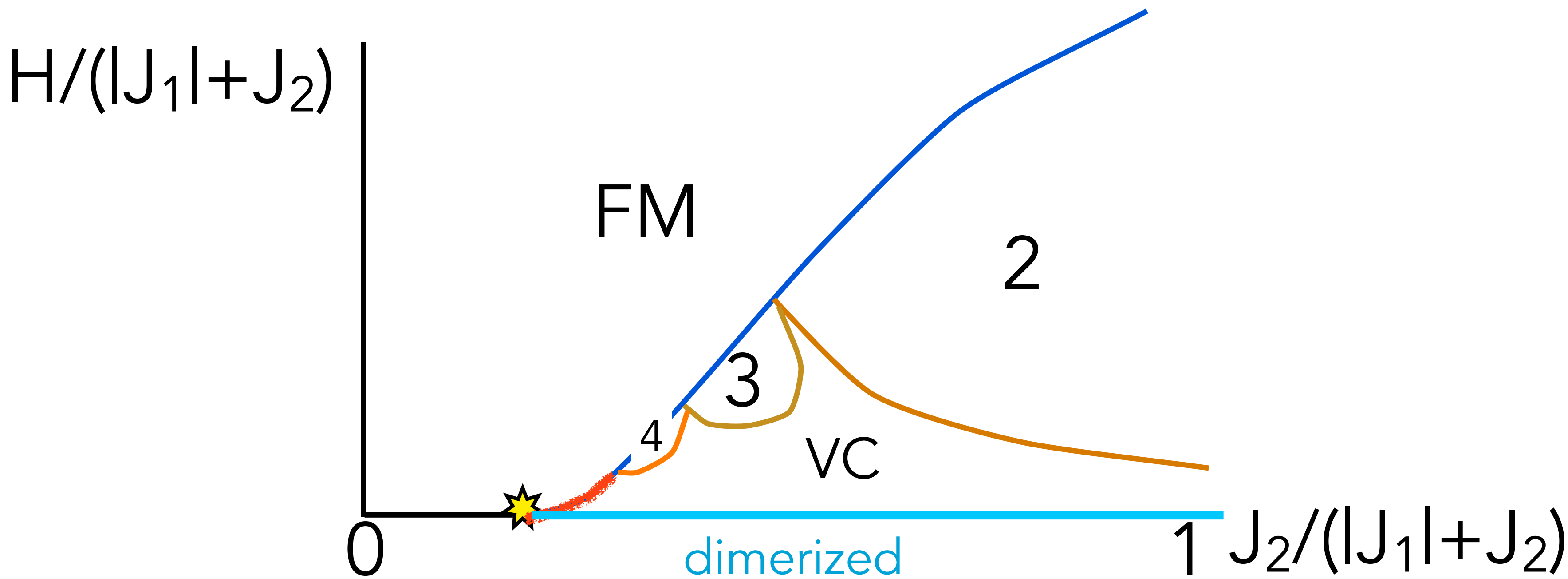
$\Pi_1(SO(3)) = Z_2$ "Z_2 vortex" instanton

S_{topo} ⟹ carries phase factor $(-1)^x$

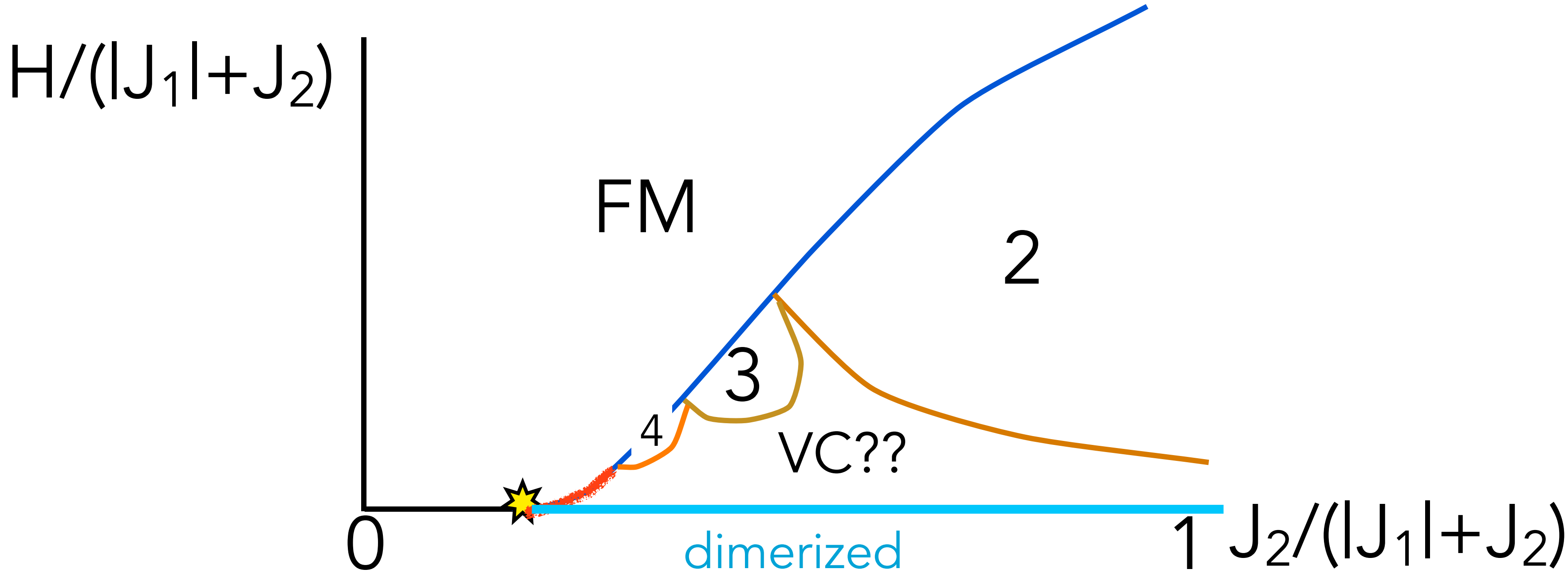
dimerization

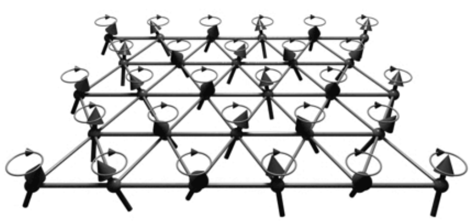


Multipolar phases



Multipolar phases





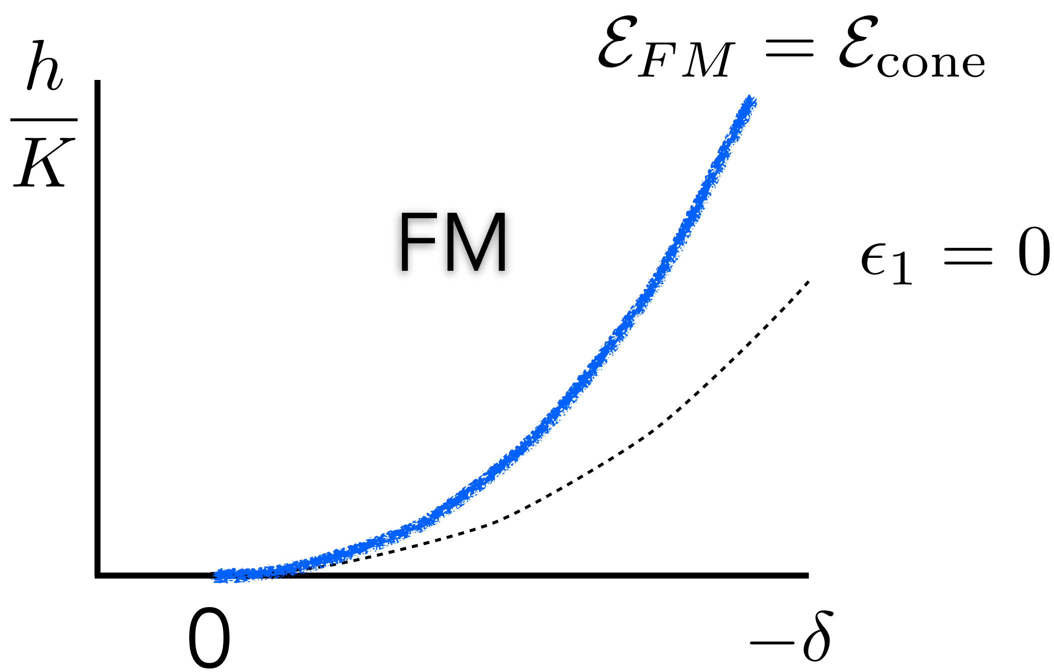
$$S_{\text{eff}} = \frac{1}{g}\int d^2x\,(\partial_\mu \phi)^2$$

c=1

$$\hat{m} = \begin{pmatrix} |\Psi|\cos(qx+\phi) \\ \pm|\Psi|\sin(qx+\phi) \\ \sqrt{1-|\Psi|^2} \end{pmatrix}$$

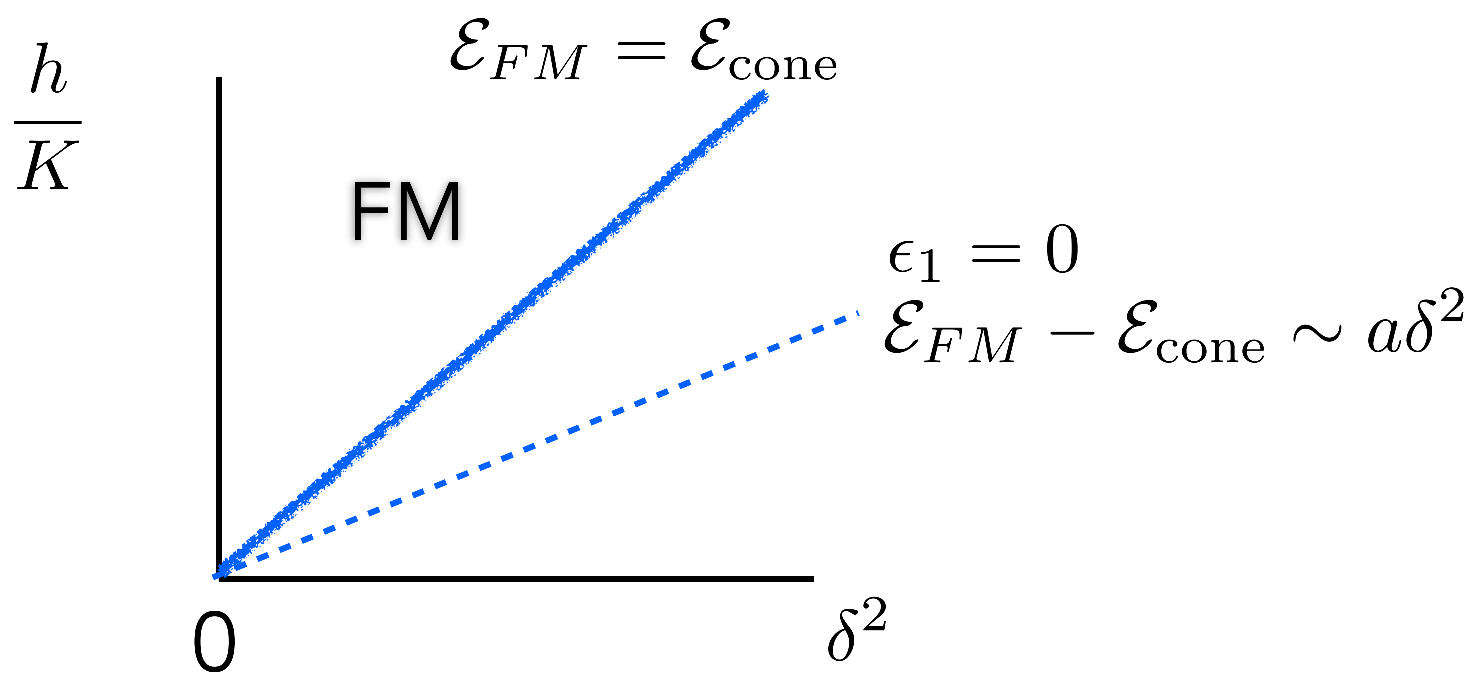
broken "TR" symmetry $\hat{z}\cdot\langle \mathbf{S}_i \times \mathbf{S}_{i+1}\rangle \neq 0$

Metamagnetic endpoint?



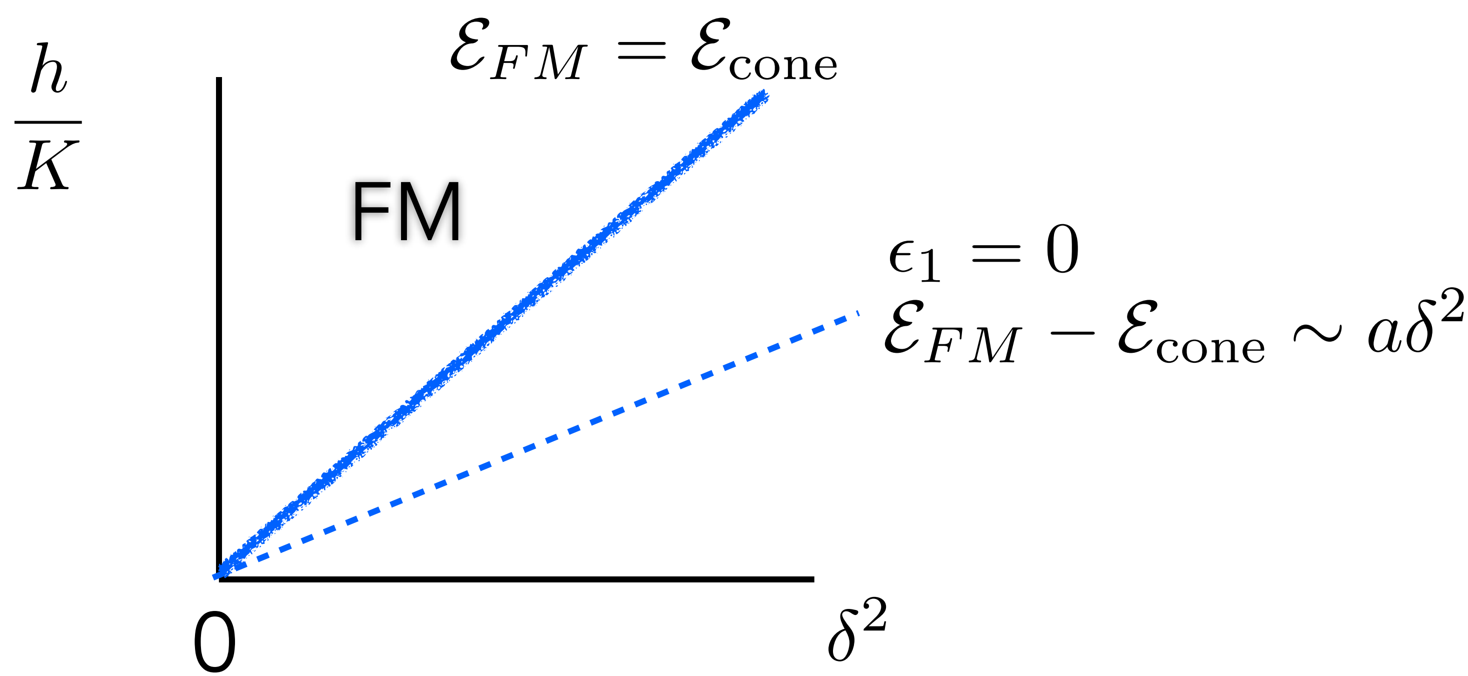
$-1 < v < -1/4$

Metamagnetic endpoint?



Quantum corrections
penalize $E_{\rm cone}$ but not $E_{\rm FM}$

Metamagnetic endpoint?



Quantum corrections
penalize E_{cone} but not E_{FM}

$$\Delta\mathcal{E}_{\rm cone} = +f(v)\delta^{5/2}$$

Metamagnetic endpoint?

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

$$\hat{m} = \sqrt{2 - n_1^2 - n_2^2}(n_1 \hat{e}_1(x) + n_2 \hat{e}_2(x)) + (1 - n_1^2 - n_2^2)\hat{e}_3(x)$$

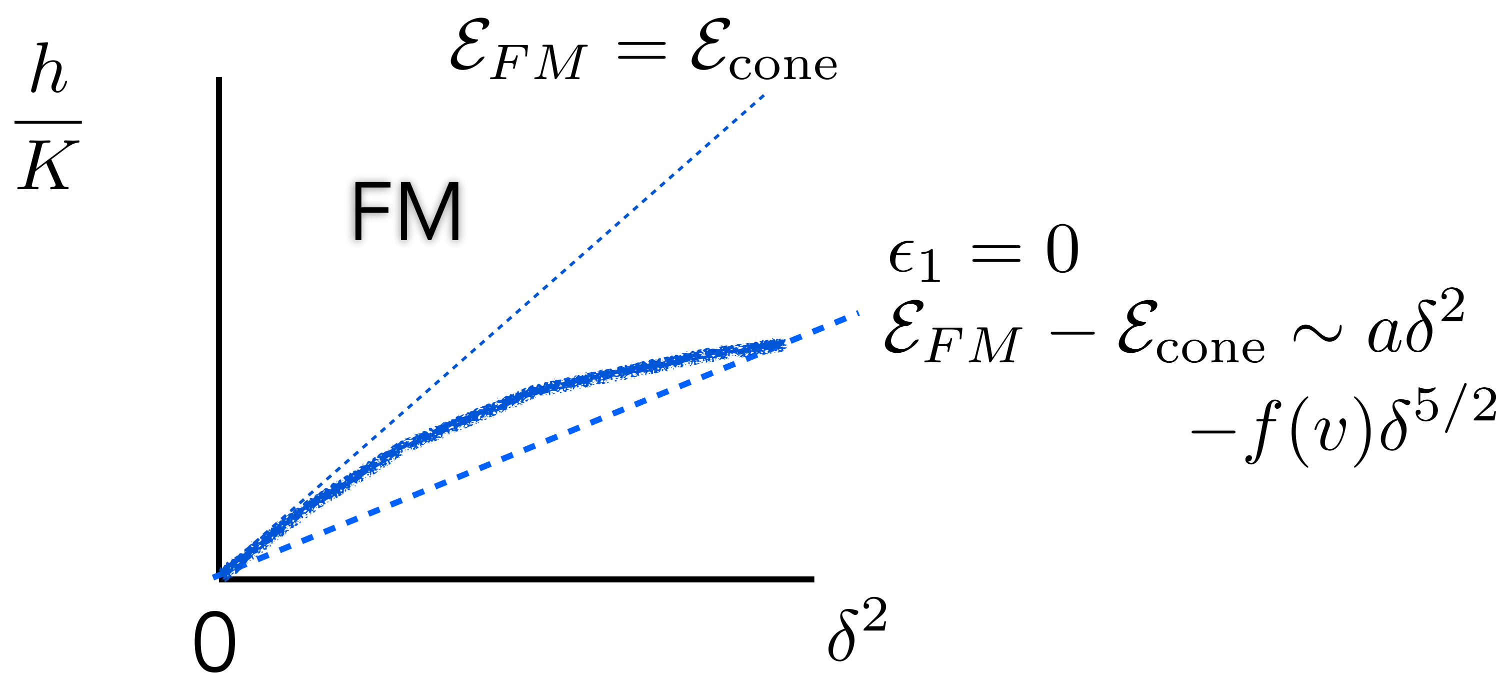
$$\hat{e}_1 \times \hat{e}_2 = \hat{e}_3 = \hat{m}^{sp}(x)$$

$$\eta = n_1 + in_2 \qquad \overline{\eta} = n_1 - in_2$$

$$S = S_{\rm sp} + \int dx\, d\tau\ \{\overline{\eta}\partial_\tau \eta + H(\overline{\eta}, \eta)\} \quad + O(\eta^3)$$

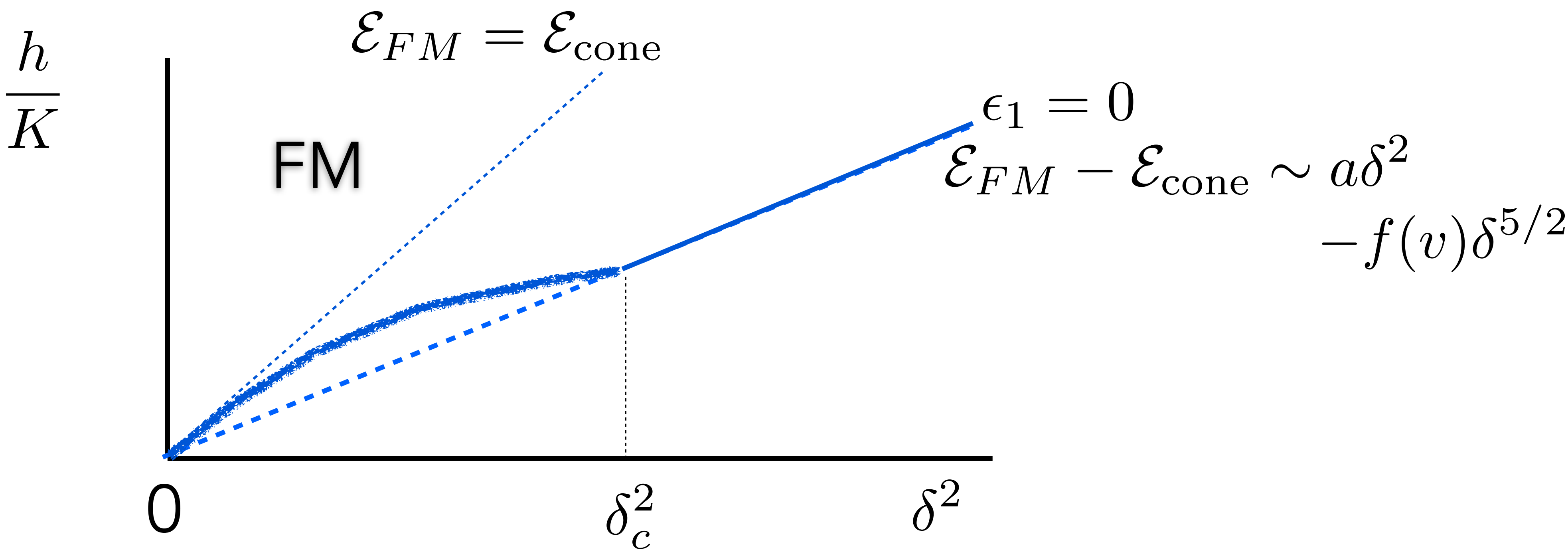
Bogoliubov transformation gives correction to GS energy

Metamagnetic endpoint?



Corrected first order curve bends slightly downward to intersect second order line

Metamagnetic endpoint?

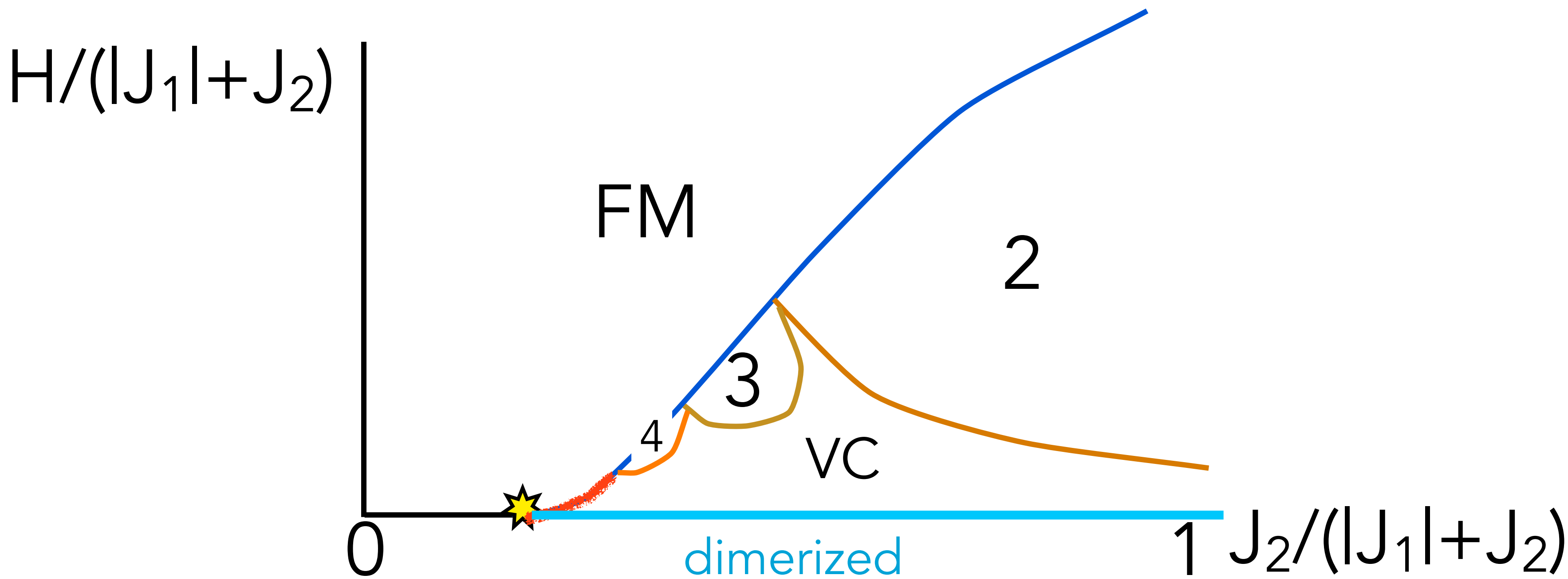


Control? $v = -1/4 - \varepsilon$

$$\mathcal{E}_{FM} - \mathcal{E}_{\text{cone}}\big|_{\epsilon_1=0} \sim \varepsilon^3\delta^2 - \varepsilon^2\delta^{5/2}$$

$$\delta_c \sim \varepsilon^2 \ll 1$$

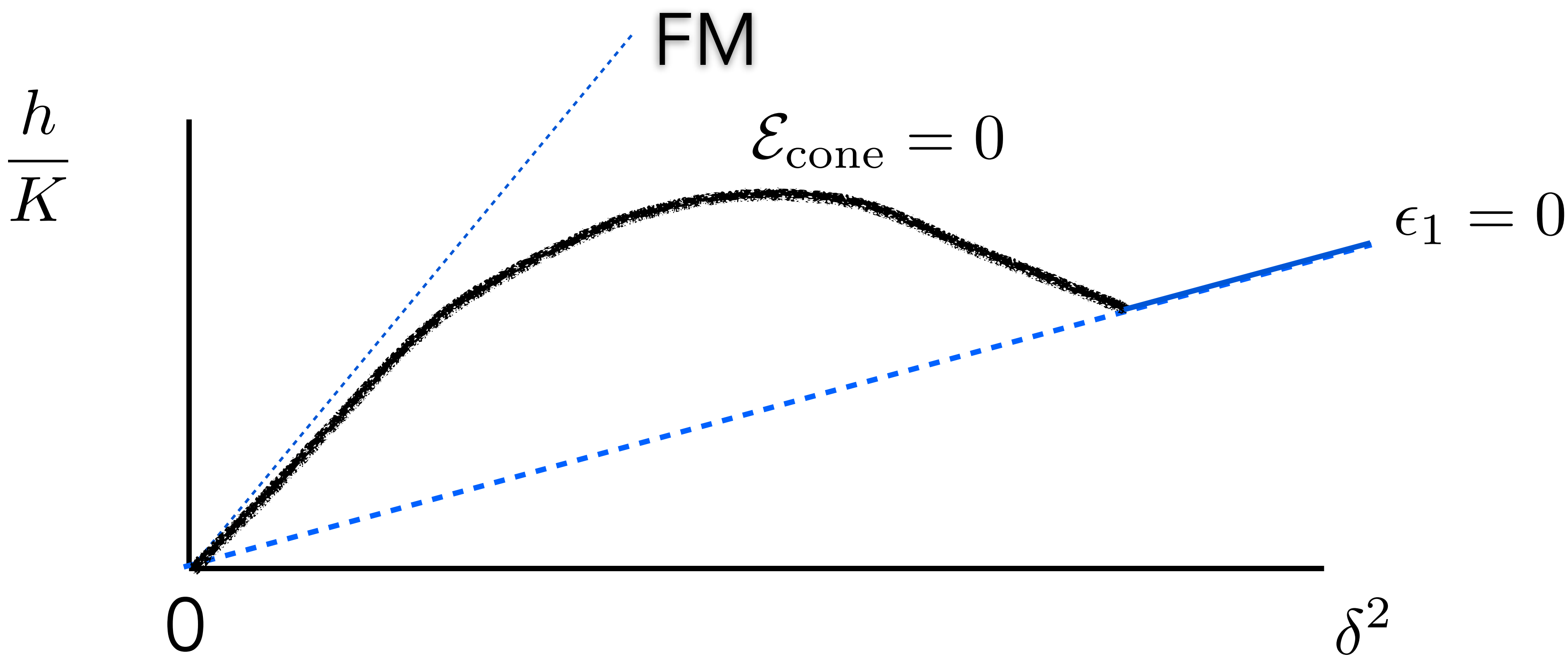
Multipolar phases



Still need to understand multipolar phases!

Instabilities

- Choose E_{FM}=0



What about multi-particle instabilities?

Low density limit

$$\hat{m}^x + i\hat{m}^y = \left(2 - \overline{\psi}\psi\right)^{1/2}\psi \qquad \hat{m}^z = 1 - \overline{\psi}\psi$$

Low energy $\quad \psi \sim \psi_1 e^{iqx} + \psi_2 e^{-iqx}$

$$\mathcal{L} \sim \overline{\psi}_a\left(\partial_\tau + h - \frac{\delta^2}{2K} - 4\delta\partial_x^2\right)\psi_a$$
$$+\gamma_1\left[\left(\overline{\psi}_1\psi_1\right)^2 + \left(\overline{\psi}_2\psi_2\right)^2\right] + \gamma_2\overline{\psi}_1\psi_1\overline{\psi}_2\psi_2$$

$$\gamma_1 = \frac{\delta^2}{4K}(1+4v) \qquad \gamma_2 = \frac{\delta^2}{K}(5+4v)$$

$$\sim -\varepsilon\delta^2 < 0 \qquad \sim +\delta^2$$

Low density limit

$$H = -4\delta \sum_i \frac{\partial^2}{\partial x_i^2} + 2\gamma_1 \sum_{i<j} \delta(x_i - x_j)$$

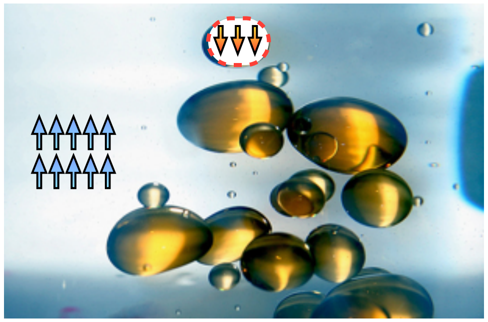
$$\gamma_1 \sim -\varepsilon\delta^2 < 0$$

attractive delta-function gas!

$$\epsilon_n = \epsilon_b \frac{n(n^2-1)}{6} \qquad \epsilon_b = -\frac{\gamma_1^2}{8\delta} = -\frac{\varepsilon^2\delta^3}{8}$$

collapse: bound states have size

$$\ell_n \sim \frac{\delta}{n|\gamma_1|} \sim \frac{1}{n\varepsilon\delta}$$



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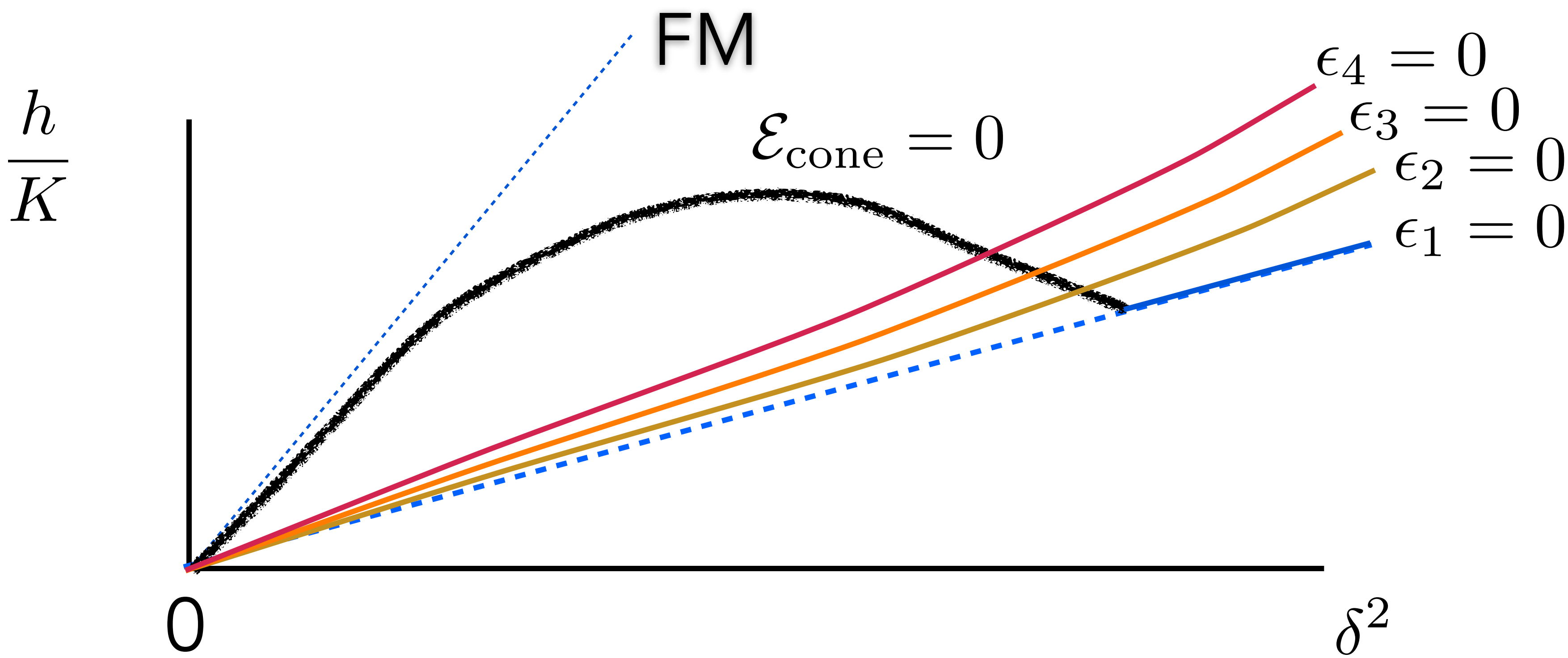

At low density level it appears higher bound state instabilities dominate

Instabilities

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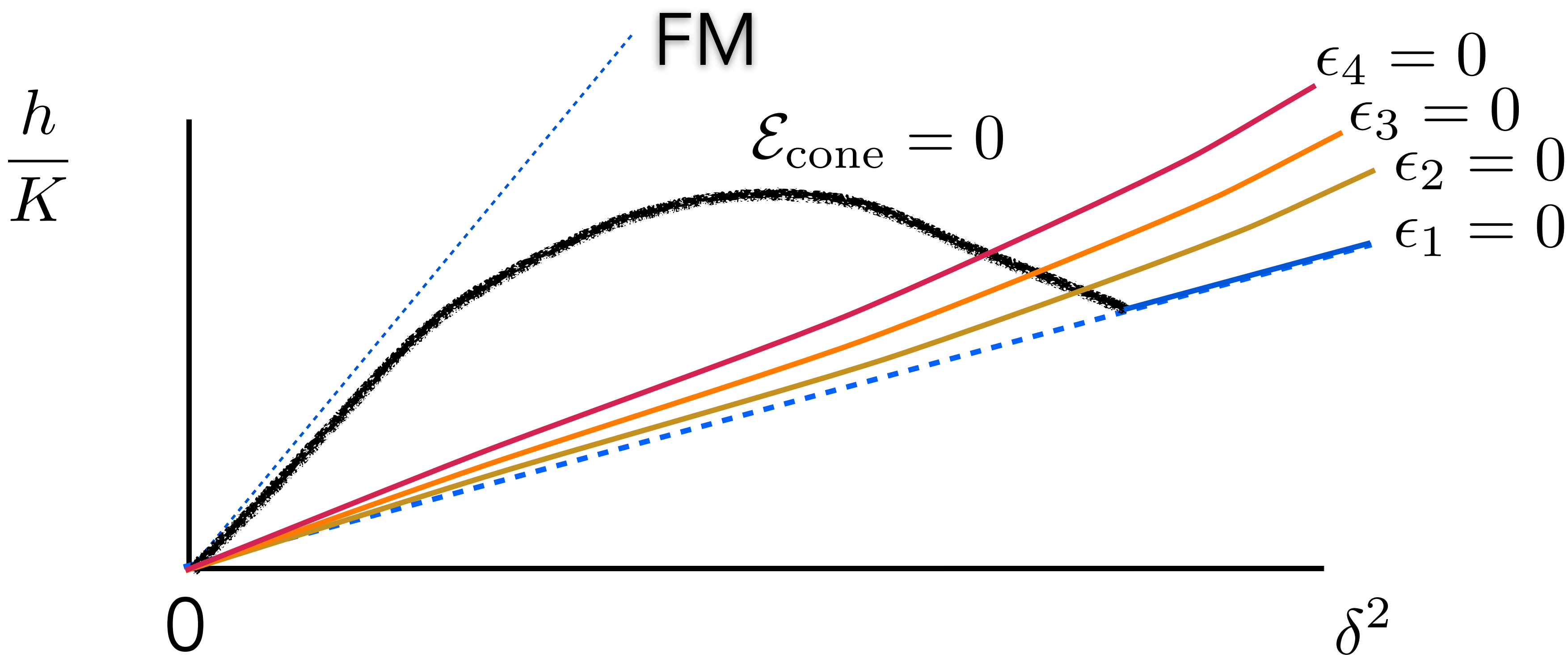

But the bound states cannot get arbitrarily deep - low density approximation is violated

A guess

- Scaling

$$\epsilon_n \sim -\varepsilon^2 \delta^3 n^3 \mathcal{F}(n\delta^{1/2}, \frac{\delta^{1/2}}{\varepsilon})$$

- Matching?

$$n\delta^{1/2} \gg 1 \qquad\qquad \mathcal{F}(X,Y) \sim 1/X^2 f(Y)$$

- Suggests maximum bound state

$$n_{\mathrm{max}} \sim \delta^{-1/2} \sim 1/\varepsilon$$

(at this scale, 3-body interactions enter)

Instabilities

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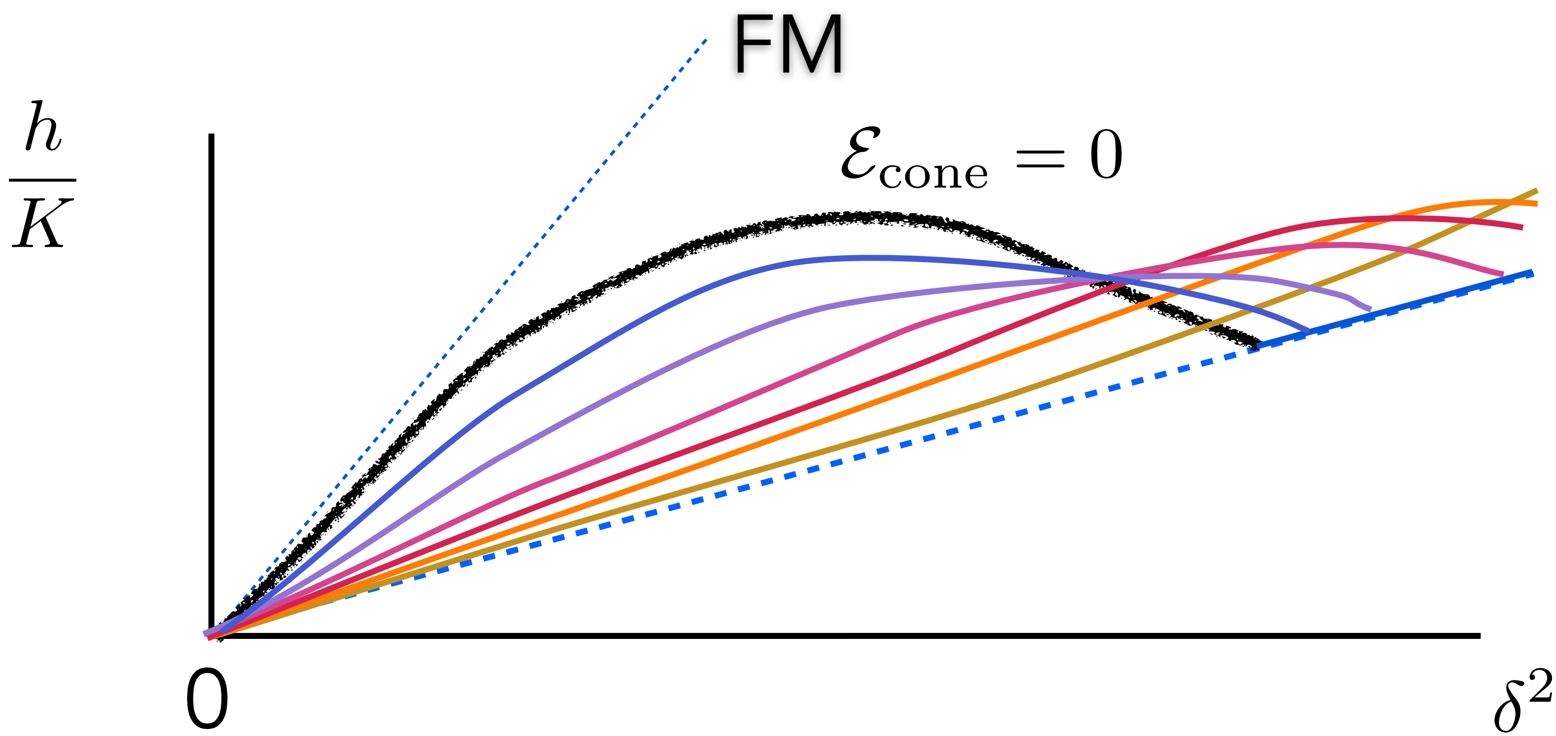

Expect that n-boson bound states bend with increasing n to approach continuum line

Instabilities

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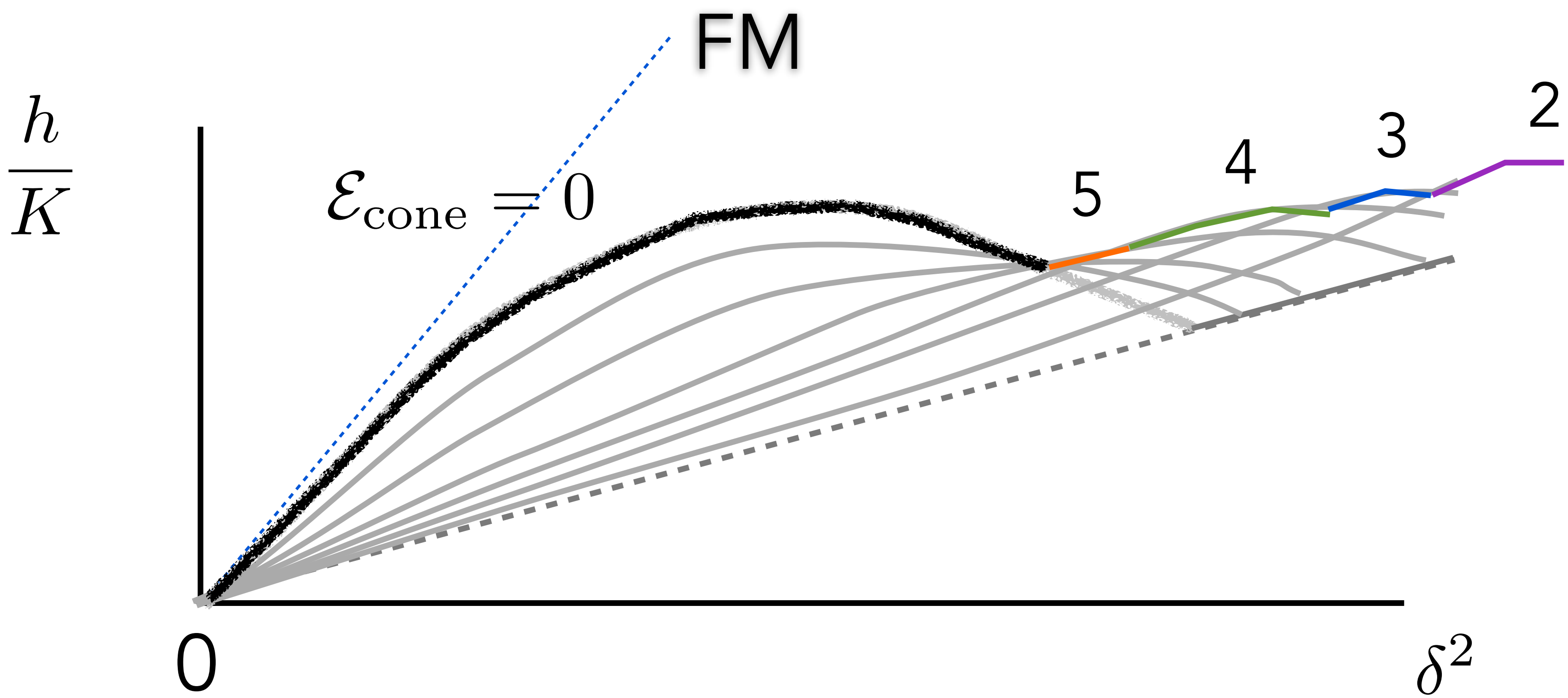

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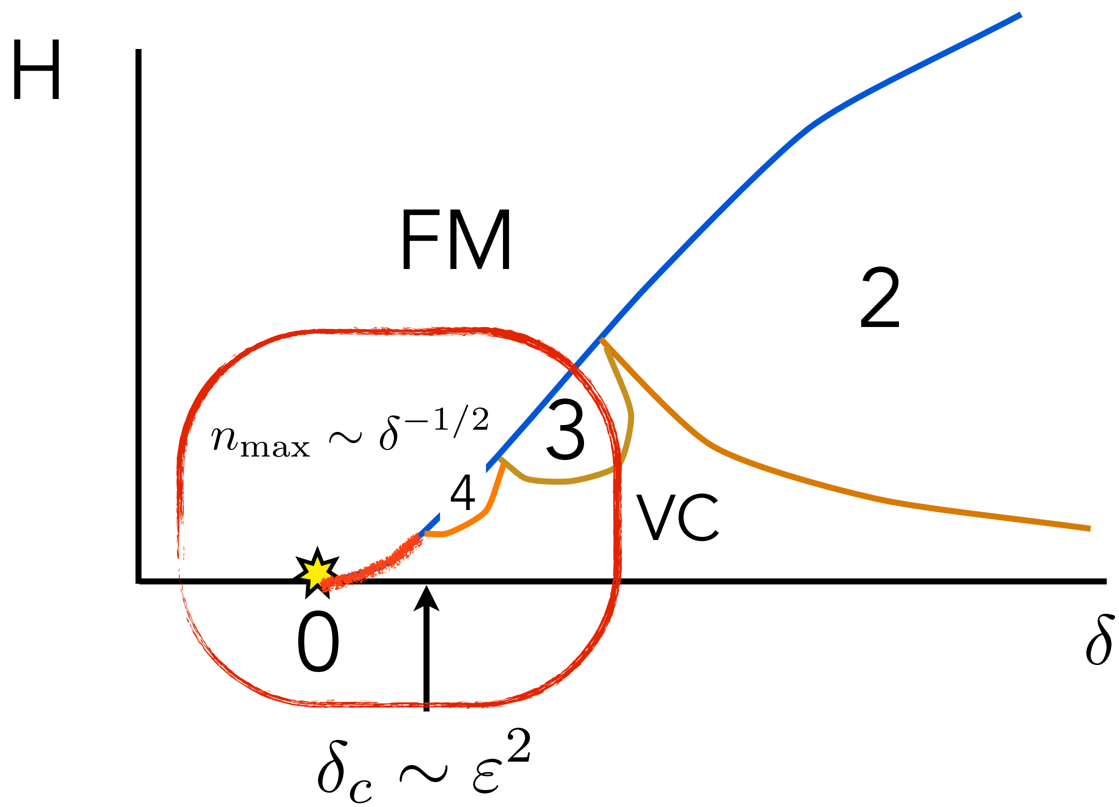
Summary



Lifshitz point is a "parent" of many phases

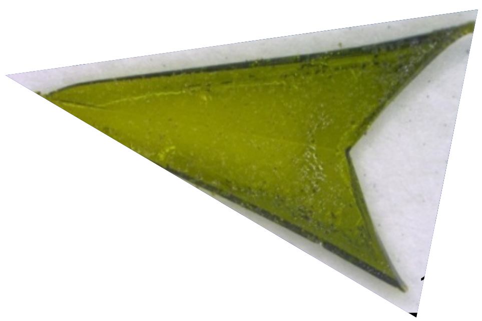
$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

Other frustrated ferromagnets

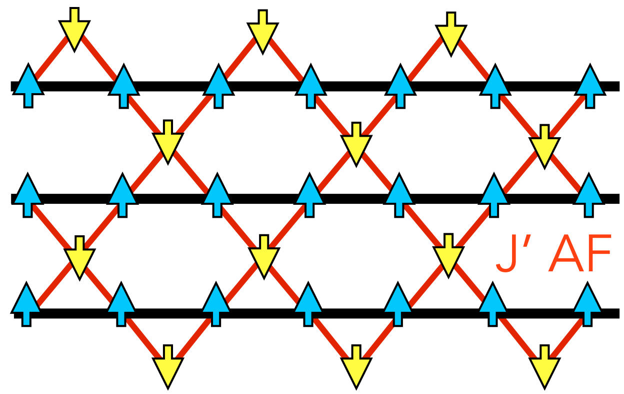
- In 1+1d, we could figure out (nearly) everything by numerically exact methods (DMRG)
- But in d>1, we have fewer tools but plenty of experiments

Eg. a frustrated ferr*i*magnet

volborthite



Hiroi group



FFM chains

$$S = \int dx d^{d-1}y d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + c|\partial_y \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

same saddle point analysis applies...