

LOCALITY AND CLASSICAL
FIELDS IN ADS,
FROM THE BOOTSTRAP

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BASED ON VARIOUS WORK WITH LIAM FITZPATRICK,
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MOTIVATION

Understand the **universal features** of AdS quantum gravity directly from the CFT:

- Fock space? AdS Locality? Long-Range Forces?
- Effective Field Theory description in AdS?
- Classical Field Configurations in AdS?
- Black Hole Microstates - Temperature, Blackness...

Which of these require **further assumptions/restrictions** beyond Conformal Symmetry and QM?

CENTRAL CFT QUESTION FOR THIS TALK

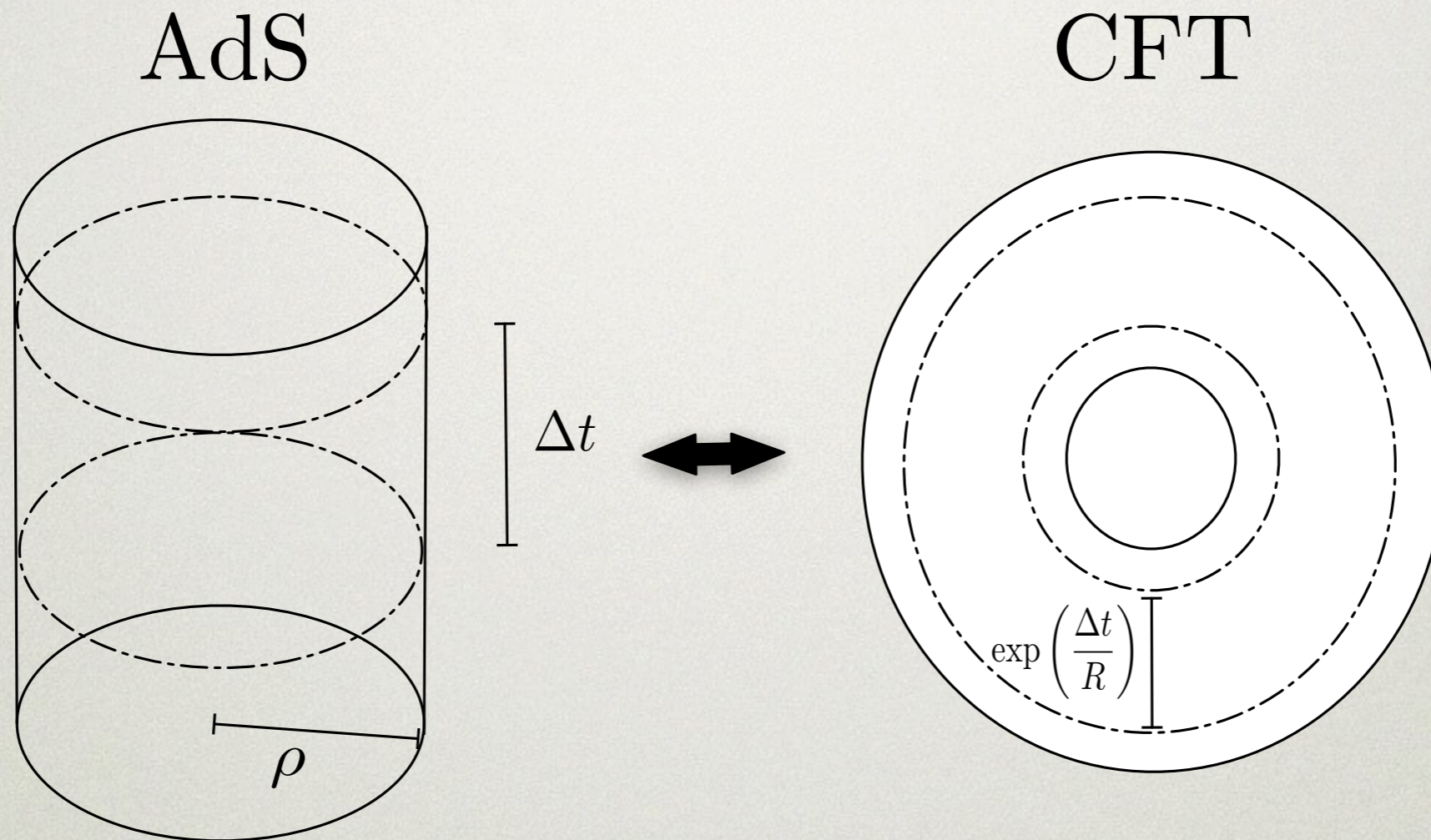
What can we say about the CFT Spectrum, OPE coefficients, and correlators assuming only that:

$$\mathcal{O}_1, \mathcal{O}_2, T \quad \text{with} \quad \mathcal{O}_i(x)\mathcal{O}_i(0) \supset T$$

Answer: Quite a bit, all of it
with an AdS Interpretation.

ADS/CFT KINEMATICS

ENERGIES AND DIMENSIONS IN ADS/CFT



$$H_{AdS} = D_{CFT}$$

REPRESENTATIONS OF GLOBAL CONFORMAL ALG

The momentum and special conformal generators act as raising and lowering operators wrt Dimension

$$[D, P_\mu] = P_\mu \quad [D, K_\mu] = -K_\mu$$

Irreducible reps built from primaries:

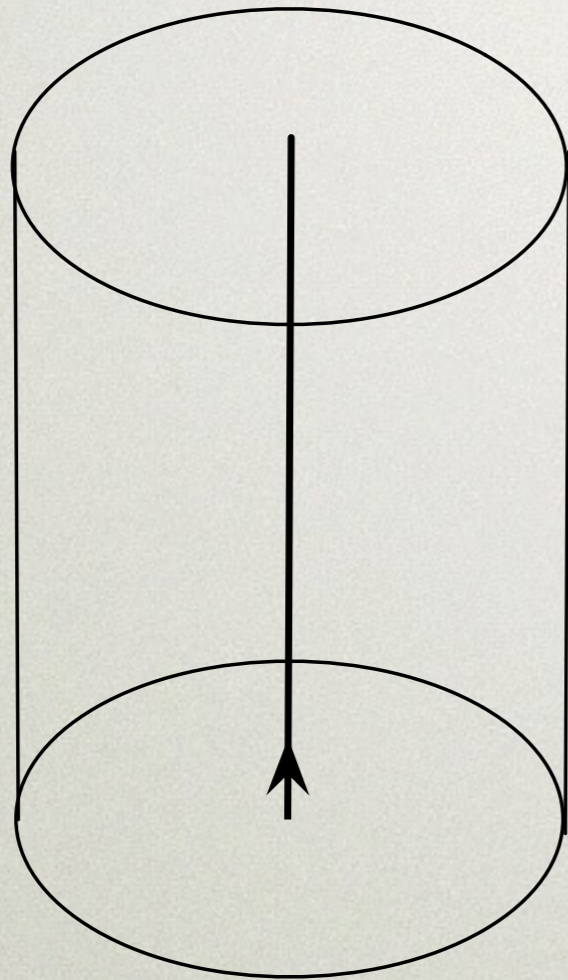
$$[K_\mu, \mathcal{O}(0)] = 0 \quad \text{or} \quad K_\mu |\psi_{\mathcal{O}}\rangle = 0$$

Can derive a unitarity relation for $\tau = \Delta - \ell$

$$\Delta_s \geq \frac{d}{2} - 1 \quad \text{and} \quad \tau_\ell \geq d - 2$$

CONFORMAL SYMMETRY AND PRIMARY STATES

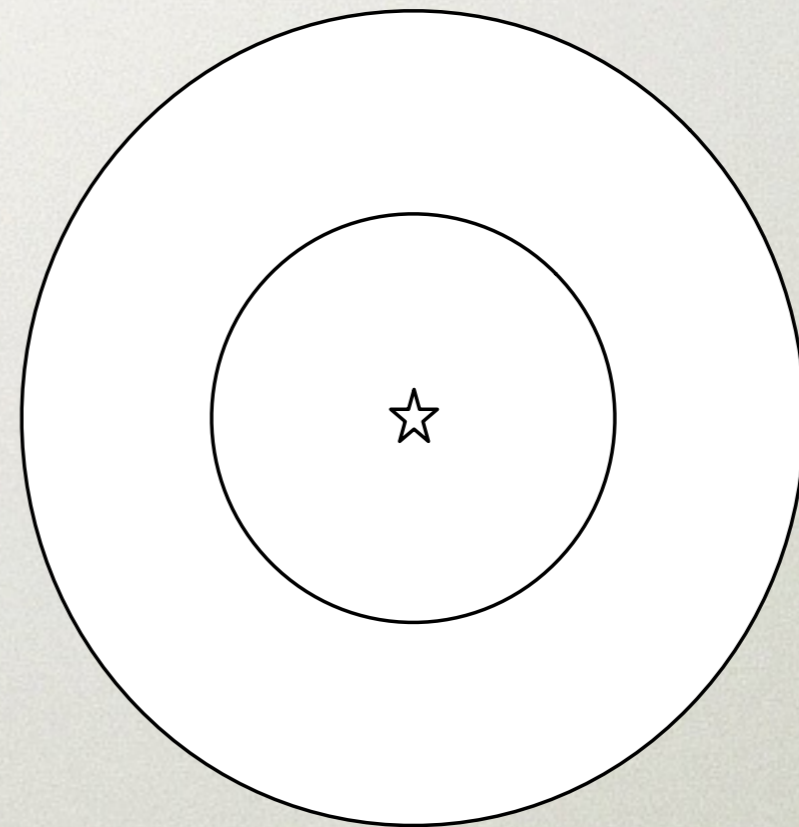
AdS



$$\psi_0(t, \rho) = e^{i\Delta t} \cos^\Delta(\rho)$$

CoM of Ground State

CFT



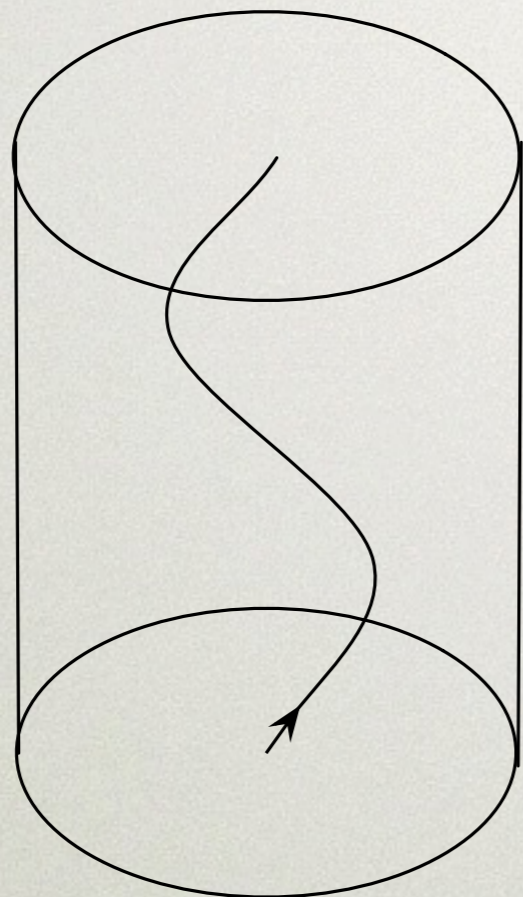
$$\mathcal{O}|0\rangle$$

CFT Primary State



EXCITED/DESCENDANT STATES

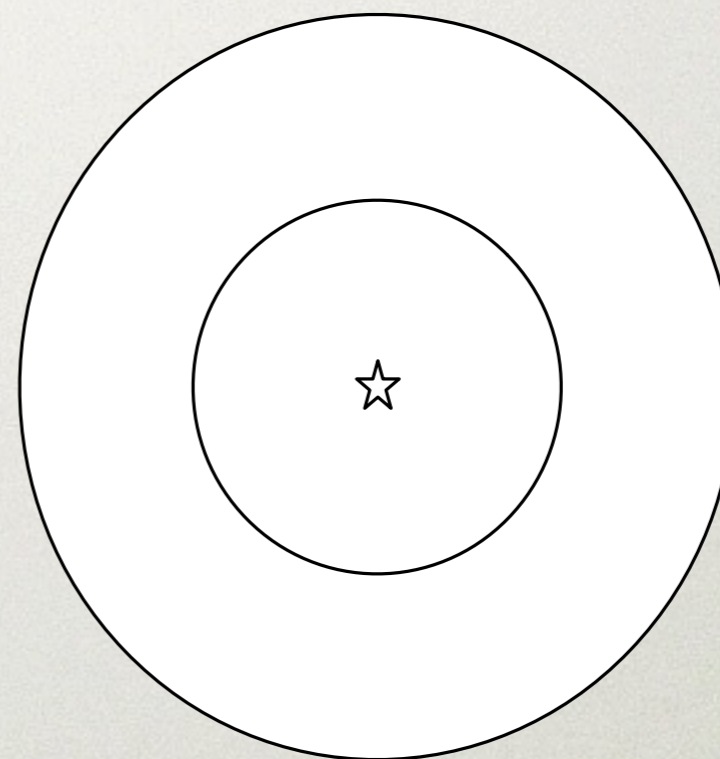
AdS



$$\psi_{n,\ell}(t, \rho, \Omega)$$

Center of Mass
for Excited State

CFT

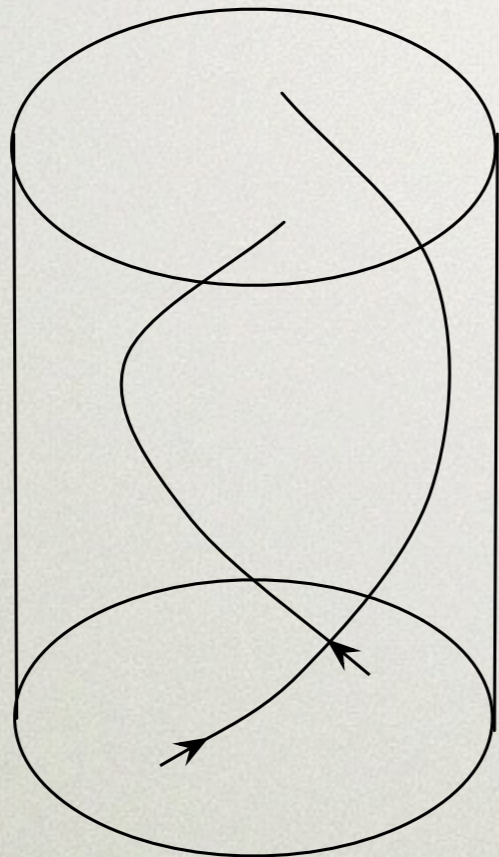


$$(\partial^{2n} \partial_{\mu_1} \cdots \partial_{\mu_\ell} \mathcal{O}) |0\rangle$$

Descendant of a
Primary

TWO PARTICLE STATES

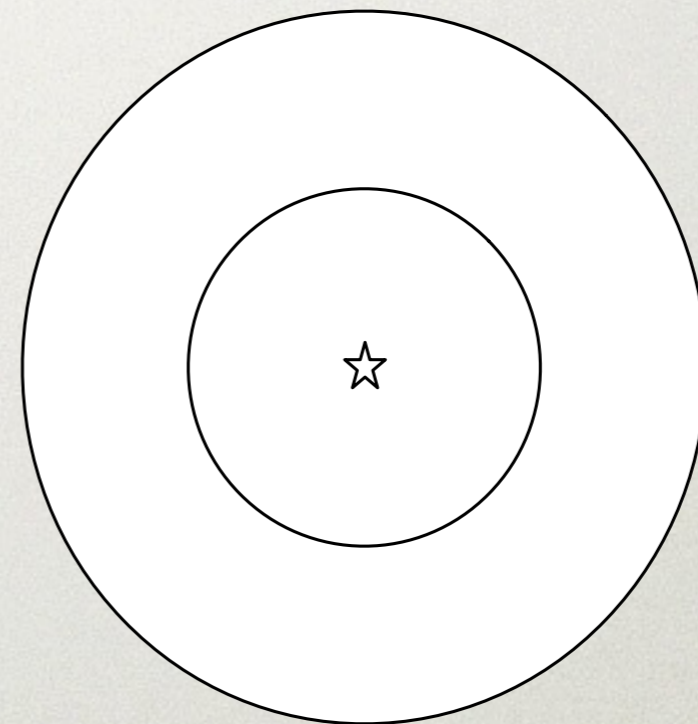
AdS



$$\psi_{n,\ell}(t_i, \rho_i, \Omega_i)$$

Two Particle State,
CoM at Origin

CFT



$$(\mathcal{O} \partial^{2n} \partial_{\mu_1} \cdots \partial_{\mu_\ell} \mathcal{O}) |0\rangle$$

'Double-Trace' Primary

**LONG-DISTANCE
ADS LOCALITY
IS COMPLETELY
UNIVERSAL**

TWO NOTIONS OF LOCALITY

AdS has an intrinsic length scale: R_{AdS}

$$ds^2 = -\cosh^2\left(\frac{\kappa}{R_{AdS}}\right) dt^2 + d\kappa^2 + \sinh^2\left(\frac{\kappa}{R_{AdS}}\right) d\Omega^2$$

Long-Distance Locality: Particles/Systems decouple when separated by $D \gg R_{AdS}$

Micro-Locality: There exists a good Effective Field Theory Description in AdS with $\Lambda_{EFT} \gg \frac{1}{R_{AdS}}$

FORMAL DEFINITION OF LONG-DISTANCE LOCALITY?

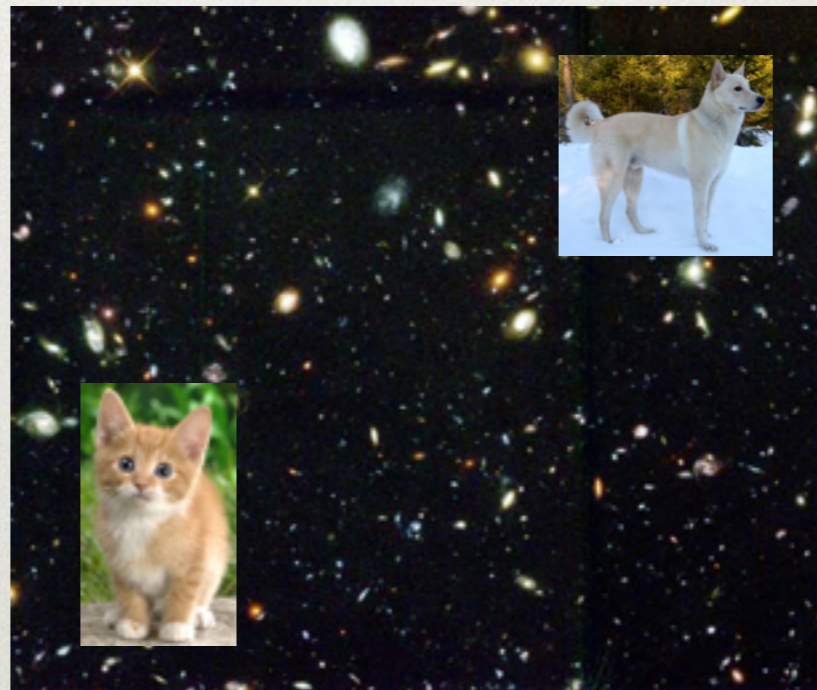
$$\psi_c =$$



$$\psi_d =$$



$$\implies \exists \psi_{cd} =$$



Statement about structure of the Hilbert Space:

$$\mathcal{H}_{AdS} = \mathcal{H}_{CFT} \approx \text{Fock Space}$$

A FOCK SPACE AT LARGE SEPARATION

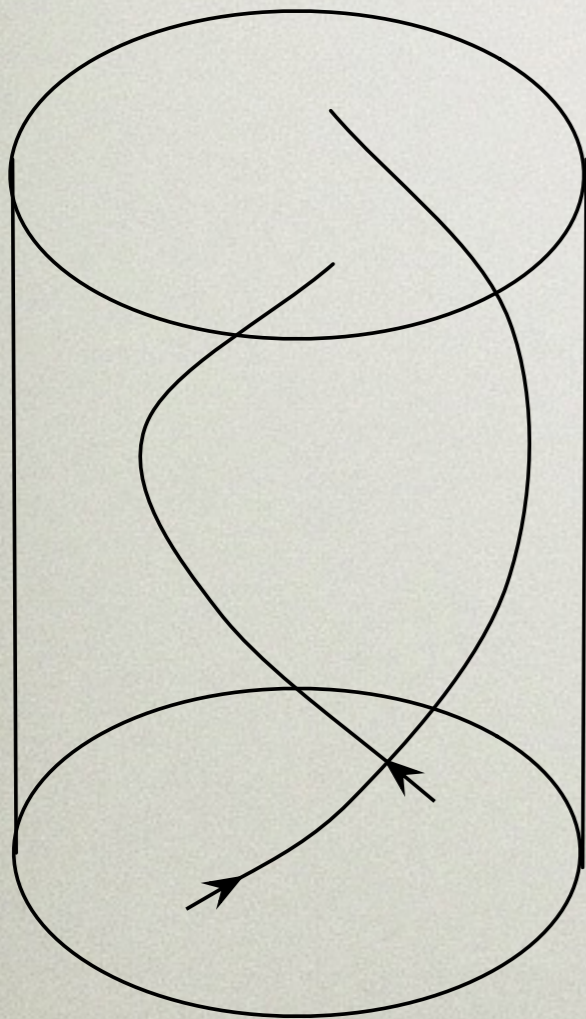


HOW TO DEFINE DISTANT FURBALLS?

Geodesic separation between cat & dog:

$$\kappa \sim R_{AdS} \log \ell$$

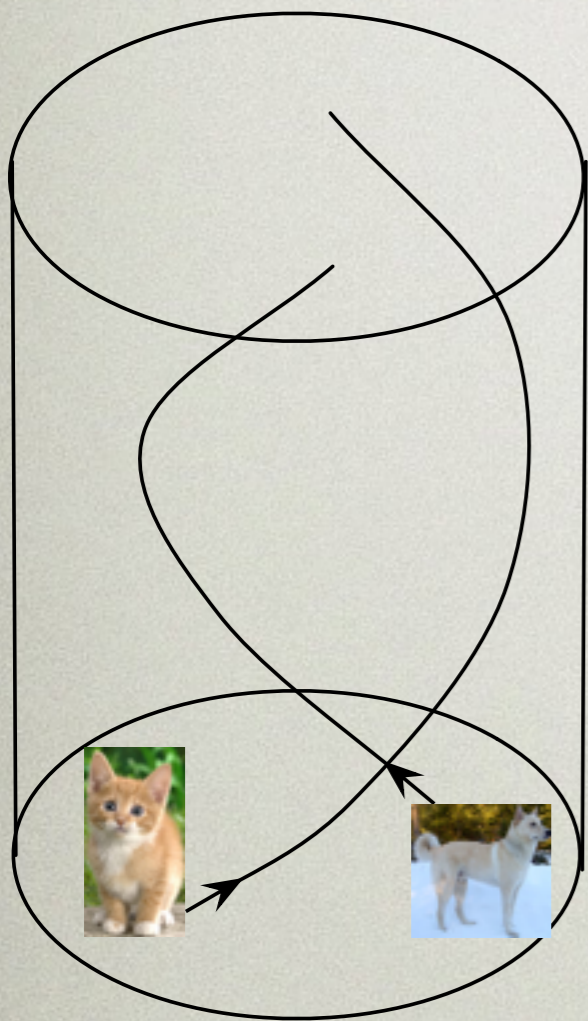
AdS



Are there two furball states at
large angular momentum???

TWO FURBALL PHYSICS?

AdS



AdS Energy = CFT Dimension:

$$E_{n\ell} = E_c + E_d + 2n + \ell + \gamma(n, \ell)$$

Anomalous dimension, $\gamma(n, \ell)$,
is a kind of 'binding energy'.

Existence of states as $\ell \rightarrow \infty$ with vanishing
 $\gamma(n, \ell)$ implies AdS Cluster Decomposition.

GENERAL THEOREM (ANY CFT IN $D > 2$)

Consider OPE of **any** two scalar primary operators:

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{\Delta, \ell} c_{\Delta, \ell}^{12} \mathcal{O}_{\Delta, \ell}(0)$$

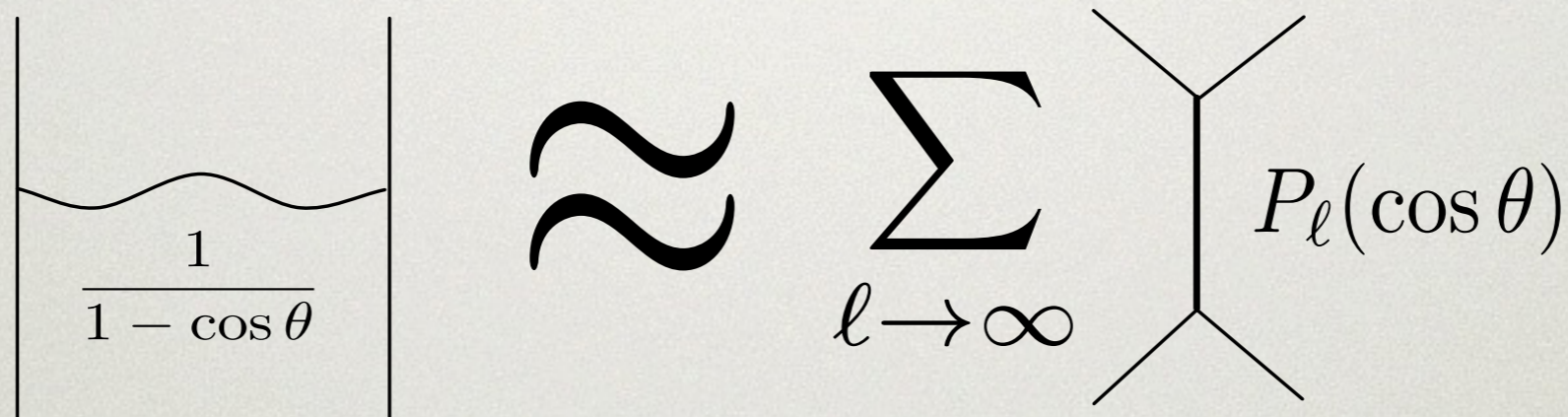
For each n there **exists** a tower of operators $\mathcal{O}_{\Delta, \ell}$ with $\Delta = \Delta_1 + \Delta_2 + 2n + \ell + \gamma(n, \ell)$ as $\ell \rightarrow \infty$, where the anomalous dimensions

$$\gamma(n, \ell) \rightarrow \frac{\gamma_n}{\ell^{\tau_m}} \quad \text{or} \quad \gamma_n e^{-\tau_m \ell}$$

from leading twist exchange, generically $T_{\mu\nu}$

THE IDEA OF THE PROOF: A SCATTERING ANALOGY

Free propagation and massless exchange
require large amplitude at large ℓ , e.g.

$$\left| \frac{1}{1 - \cos \theta} \right| \approx \sum_{\ell \rightarrow \infty} \text{Diagram} P_{\ell}(\cos \theta)$$
The diagram shows a mathematical equation with a diagrammatic interpretation. On the left, a vertical line is connected to another vertical line by a wavy line representing a propagator. Below the wavy line is the fraction $\frac{1}{1 - \cos \theta}$. This is followed by a double wavy line symbol \approx . To the right of the symbol is a summation $\sum_{\ell \rightarrow \infty}$. Next to the summation is a diagram of a vertical line with two diagonal lines branching out from its top and bottom, representing a partial wave. To the right of this diagram is the label $P_{\ell}(\cos \theta)$.

Completely analogous CFT phenomenon.

Implies existence of large ℓ states.

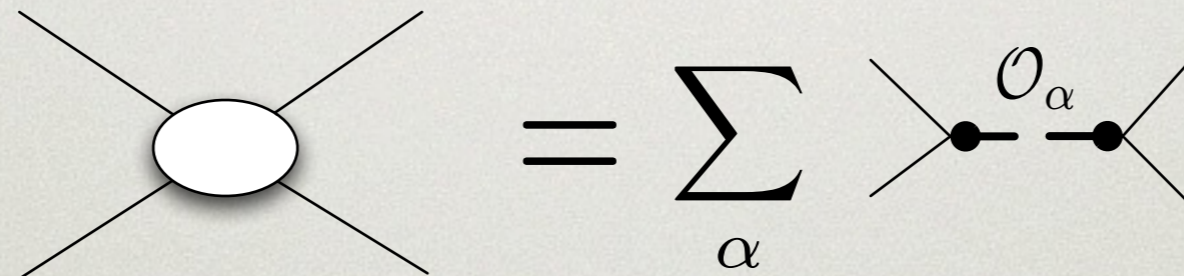
Partial Wave Amplitudes \rightarrow Conformal Partial Waves

THE CONFORMAL PARTIAL WAVE DECOMPOSITION

Insert 1, organize according to conformal symmetry:

$$\langle \mathcal{O}_1 \mathcal{O}_2 \left(\sum_{\alpha} |\alpha\rangle \langle \alpha| \right) \mathcal{O}_3 \mathcal{O}_4 \rangle$$

Since **operators = states** in the CFT, write 4-pt as



The diagram shows a four-point correlator on the left, represented by a white oval with four external lines. This is equal to a sum over α of a product of two three-point correlators on the right. Each three-point correlator is shown as a vertex with two external lines and one internal line connecting the two vertices. The internal line is labeled \mathcal{O}_α .

A sum over exchange labeled by **primary operators**,
magnitude given by product of 3-pt correlators.

LIGHT-CONE OPE LIMIT (DIAGRAMS = BLOCKS)

The equivalent of a t-channel singularity in the CFT is the light-cone OPE limit:

$$\langle \mathcal{O}_1 \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_2 \rangle = u^{-\frac{\Delta_1 + \Delta_2}{2}} \left(1 + u^{\frac{\tau_T}{2}} f_T(v) + \dots \right)$$

giving a bootstrap equation

$$\begin{array}{c} \mathcal{O}_1 \\ | \\ \text{---} \mathbf{1} \text{---} \\ | \\ \mathcal{O}_1 \end{array}
 +
 \begin{array}{c} \mathcal{O}_1 \\ | \\ \text{---} T \text{---} \\ | \\ \mathcal{O}_1 \end{array}
 \approx
 \sum_{l \rightarrow \infty}
 \begin{array}{c} \mathcal{O}_1 \quad \mathcal{O}_2 \\ \diagdown \quad \diagup \\ | \\ \text{---} [\mathcal{O}_1 \mathcal{O}_2]_{n,l} \text{---} \\ \diagup \quad \diagdown \\ \mathcal{O}_1 \quad \mathcal{O}_2 \end{array}$$

Example: long-range gravity is completely universal.

MICRO-LOCALITY
FOR
SPECIAL CFTS

MICRO-LOCALITY (OR FLAT SPACE LOCALITY)

Showed that **All CFTs** have a Fock Space spectrum at long-distances. Would need at **short-distances** too.

To make progress, need **perturbative** expansion.
(Our definition is in terms of AdS EFT.)

Both are **special** features, invalid for most CFTs.

MICRO-LOCALITY (OR FLAT SPACE LOCALITY)

Need a 3rd special feature. Counter-example:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{N^2} [\lambda\phi^4 + R_{AdS}^4(\partial\phi)^4 + \dots]$$

To see what went wrong, look at flat space S-Matrix:

$$\mathcal{A} = \frac{1}{N^2} (\lambda + R^4(s^2 + t^2 + u^2) + R^8 s^4 + R^{12} S^6 + \dots)$$

Breaks down at the scale R , despite weak coupling.

EFTs well-approximated by polynomial Amplitudes.

MELLIN SPACE

AS ADS MOMENTUM SPACE

Can define Mellin Amplitude for CFT correlator:

$$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_4) \rangle \propto \int \frac{ds dt}{(2\pi i)^2} \tilde{M}(s, t) u^{-s} v^{-t}$$

Mellin variables direct analog of Mandelstam s and t .

Mellin Amplitude is 'momentum space' for AdS.

Asymptotic bounds \longrightarrow EFT description in AdS.

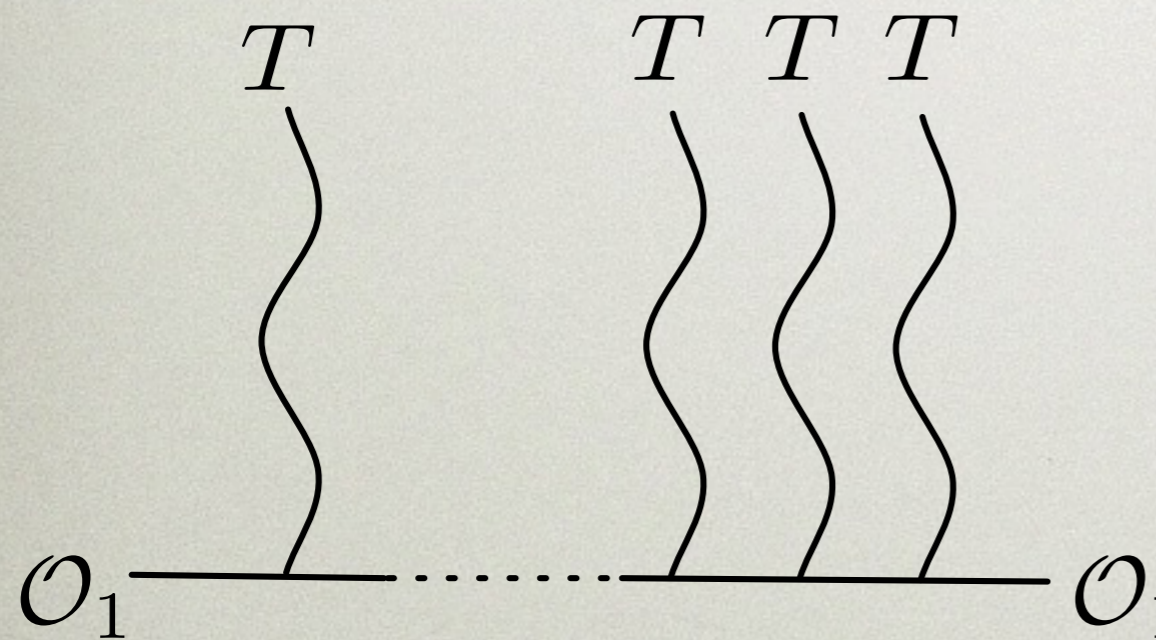
CLASSICAL ADS

FIELDS AND

'EIKONALIZATION'

MULTIPLE EXCHANGES AND CLASSICAL FIELDS

In any spacetime, effective classical fields emerge from the summation of infinite series of simple diagrams, such as:



The diagram illustrates the summation of multiple exchanges between two sources, \mathcal{O}_1 . On the left, a single wavy line labeled T connects two \mathcal{O}_1 sources. To its right, a dotted line indicates a continuation of the series. Further right, three wavy lines labeled T connect the two \mathcal{O}_1 sources, representing multiple exchanges. An arrow points from these diagrams to the resulting potential function:

$$\implies V(r) = \frac{GM}{r^{d-2}}$$

Can get Schwarzschild by including interactions.

INTER-RELATED QUESTIONS:

Does the Bootstrap know about the multiple exchanges that build up a classical field in AdS?

Why are Mellin amplitude asymptotics important?

Can we take $\frac{1}{\ell}$ perturbation theory further?

How far can we go with no assumption except:

$$\mathcal{O}_1, \mathcal{O}_2, T \quad \text{with} \quad \mathcal{O}_i(x)\mathcal{O}_i(0) \supset T$$

A BOOTSTRAP IMPLICATION

$$\begin{array}{c} \mathcal{O}_1 \\ | \\ \text{---} \mathbb{1} \text{---} \\ | \\ \mathcal{O}_2 \end{array} + \begin{array}{c} \mathcal{O}_1 \\ | \\ \text{---} T \text{---} \\ | \\ \mathcal{O}_2 \end{array} + \begin{array}{c} \mathcal{O}_1 \\ | \\ \text{---} \sum_{n,\ell} [TT]_{n,\ell} \text{---} \\ | \\ \mathcal{O}_2 \end{array} + \dots \approx \sum_{\ell \rightarrow \infty} \begin{array}{c} \mathcal{O}_1 \quad \mathcal{O}_2 \\ \diagdown \quad \diagup \\ | \\ [O_1 O_2]_{n,\ell} \\ \diagup \quad \diagdown \\ \mathcal{O}_1 \quad \mathcal{O}_2 \end{array}$$

On the right-hand side, $[O_1 O_2]_{n,\ell}$ anomalous dim:

$$g_{\tau,\ell}(v, u) \approx \left(1 + \frac{\gamma_n}{2\ell^{\tau_m}} \ln v + \frac{\gamma_n^2}{8\ell^{2\tau_m}} \ln^2 v + \dots \right) g_{\tau_n,\ell}(v, u)$$

How can we match this on the left-hand side?

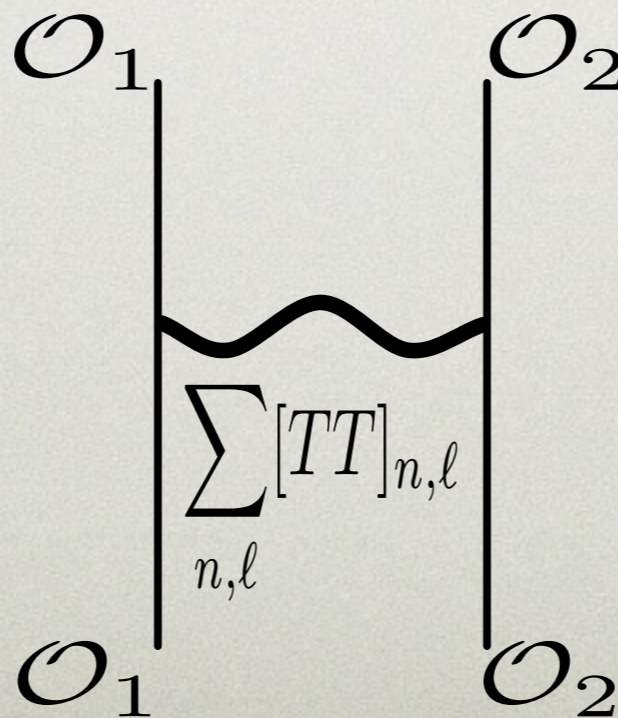
Not obvious because Bootstrap is 'on-shell'.

BE FRUITFUL AND MULTIPLY...

Fock Space Theorem implies existence of:

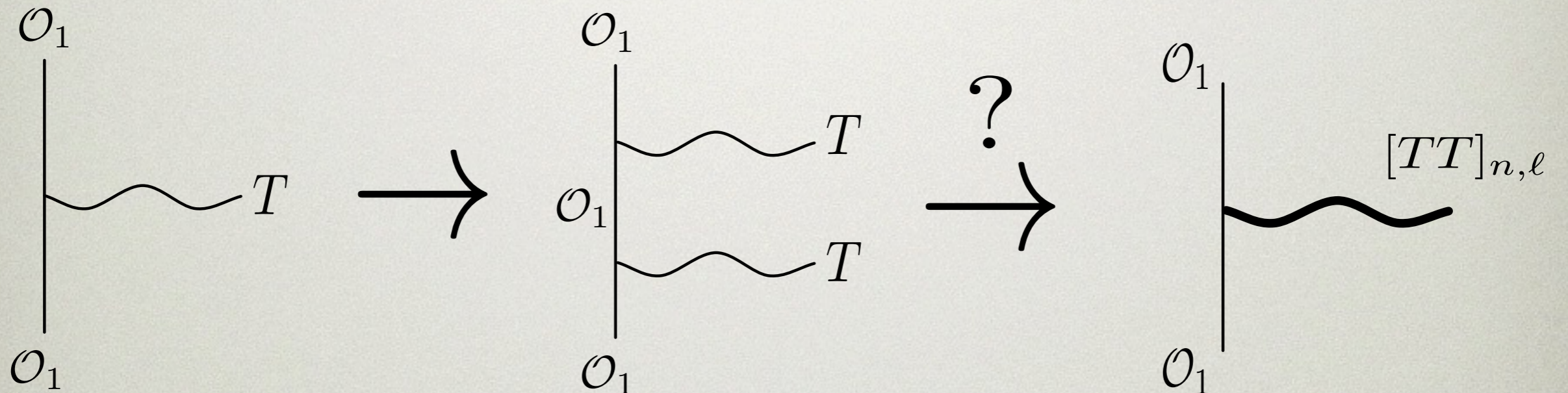
$$[\mathcal{O}_1\mathcal{O}_2]_{n,\ell}, [\mathcal{O}_1T]_{n,\ell}, [TT]_{n,\ell}, \dots, [[\mathcal{O}_1\mathcal{O}_1]_{n,\ell}T]_{n',\ell'}, \dots$$

So at large spin, we are allowed to ask about:



A NEEDED OPE LIMIT

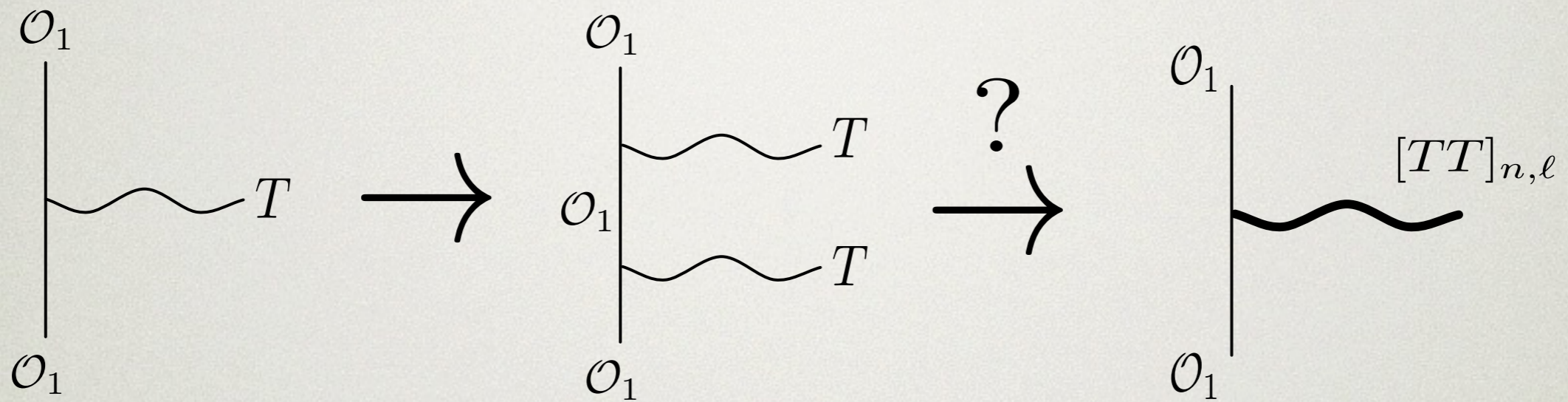
We would like to use the OPE to deduce at large spin:



Without further assumptions, ill-defined!!

Cross-channel OPE limits of Blocks do not exist.

A NEEDED OPE LIMIT



Need to expand a function such as

$${}_2F_1(1, 1, 2, z) = -\frac{\log(1-z)}{z}$$

Near $z = 1$

One reason why crossing symmetry so non-trivial!

NEEDED OPE LIMIT, UNDERSTOOD FROM MELLIN

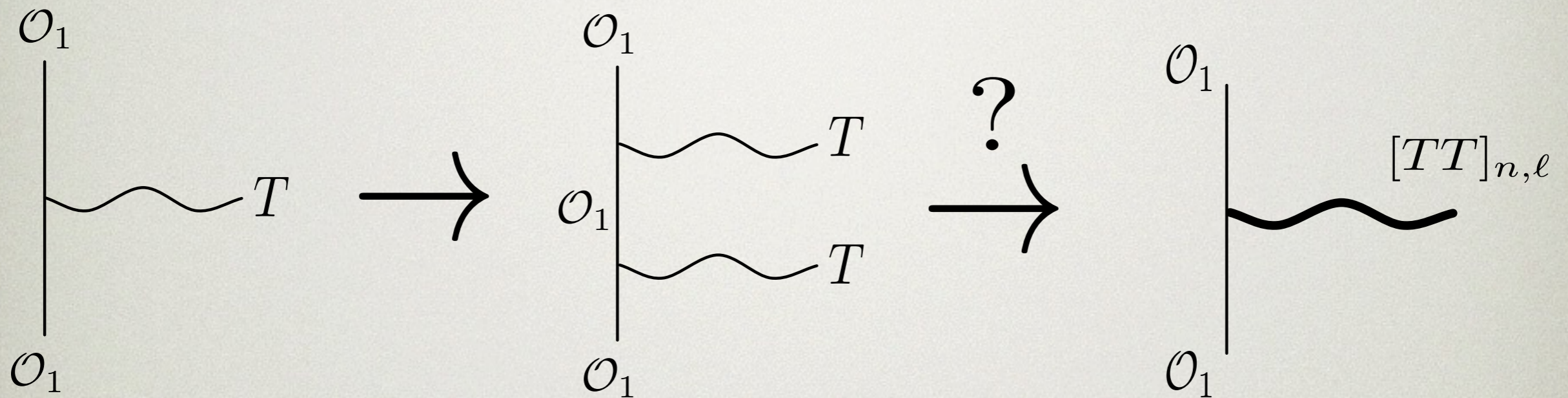
Toy example:
$$G(z) = \int_{-i\infty}^{i\infty} M(\delta) z^{-\delta} d\delta$$

Both OPE limits, $z = 0$ and $z = 1$ can be studied simultaneously.

Behavior near $z = 0$ determined by poles.

Behavior near $z = 1$ determined by asymptotics.

A NEEDED OPE LIMIT, UNDERSTOOD VIA MELLIN



Pathology due to bad asymptotic behavior.

Result controlled by leading Mellin pole,
which gives the **universal behavior** expected
from the bootstrap equation.

FOR ANY CFT, CONFORMAL BLOCKS CAN BE SUMMED

If Mellin Amplitude is exponentially bounded, we can sum or 'Eikonalize' the conformal blocks:

$$\begin{array}{c} \mathcal{O}_1 \\ | \\ \text{---} \mathbf{1} \text{---} \\ | \\ \mathcal{O}_2 \end{array} + \begin{array}{c} \mathcal{O}_1 \\ | \\ \text{---} T \text{---} \\ | \\ \mathcal{O}_2 \end{array} + \begin{array}{c} \mathcal{O}_1 \\ | \\ \text{---} \sum_{n,l} [TT]_{n,l} \text{---} \\ | \\ \mathcal{O}_2 \end{array} + \dots \approx \sum_{l \rightarrow \infty} \begin{array}{c} \mathcal{O}_1 \quad \mathcal{O}_2 \\ \diagdown \quad \diagup \\ | \\ \mathcal{O}_1 \quad \mathcal{O}_2 \end{array} [O_1 O_2]_{n,l}$$

AdS QFT aka 1/N Perturbation Theory is a special case.

FOR ANY CFT, CONFORMAL BLOCKS CAN BE SUMMED

$$\begin{array}{c} \mathcal{O}_1 \\ | \\ \text{---} \mathbf{1} \text{---} \\ | \\ \mathcal{O}_1 \end{array}
 \begin{array}{c} \mathcal{O}_2 \\ | \\ \text{---} \\ | \\ \mathcal{O}_2 \end{array}
 +
 \begin{array}{c} \mathcal{O}_1 \\ | \\ \text{---} \text{---} T \text{---} \\ | \\ \mathcal{O}_1 \end{array}
 \begin{array}{c} \mathcal{O}_2 \\ | \\ \text{---} \\ | \\ \mathcal{O}_2 \end{array}
 +
 \begin{array}{c} \mathcal{O}_1 \\ | \\ \text{---} \text{---} \sum_{n,l} [TT]_{n,l} \text{---} \\ | \\ \mathcal{O}_1 \end{array}
 \begin{array}{c} \mathcal{O}_2 \\ | \\ \text{---} \\ | \\ \mathcal{O}_2 \end{array}
 + \dots \approx \sum_{l \rightarrow \infty} \begin{array}{c} \mathcal{O}_1 \quad \mathcal{O}_2 \\ \diagdown \quad \diagup \\ \text{---} \\ \diagup \quad \diagdown \\ \mathcal{O}_1 \quad \mathcal{O}_2 \end{array} [O_1 O_2]_{n,l}$$

In the limit $\tau_T \ll \Delta_1, \Delta_2$

$$e^{c_{11}^T c_{22}^T g_T(u,v)}$$

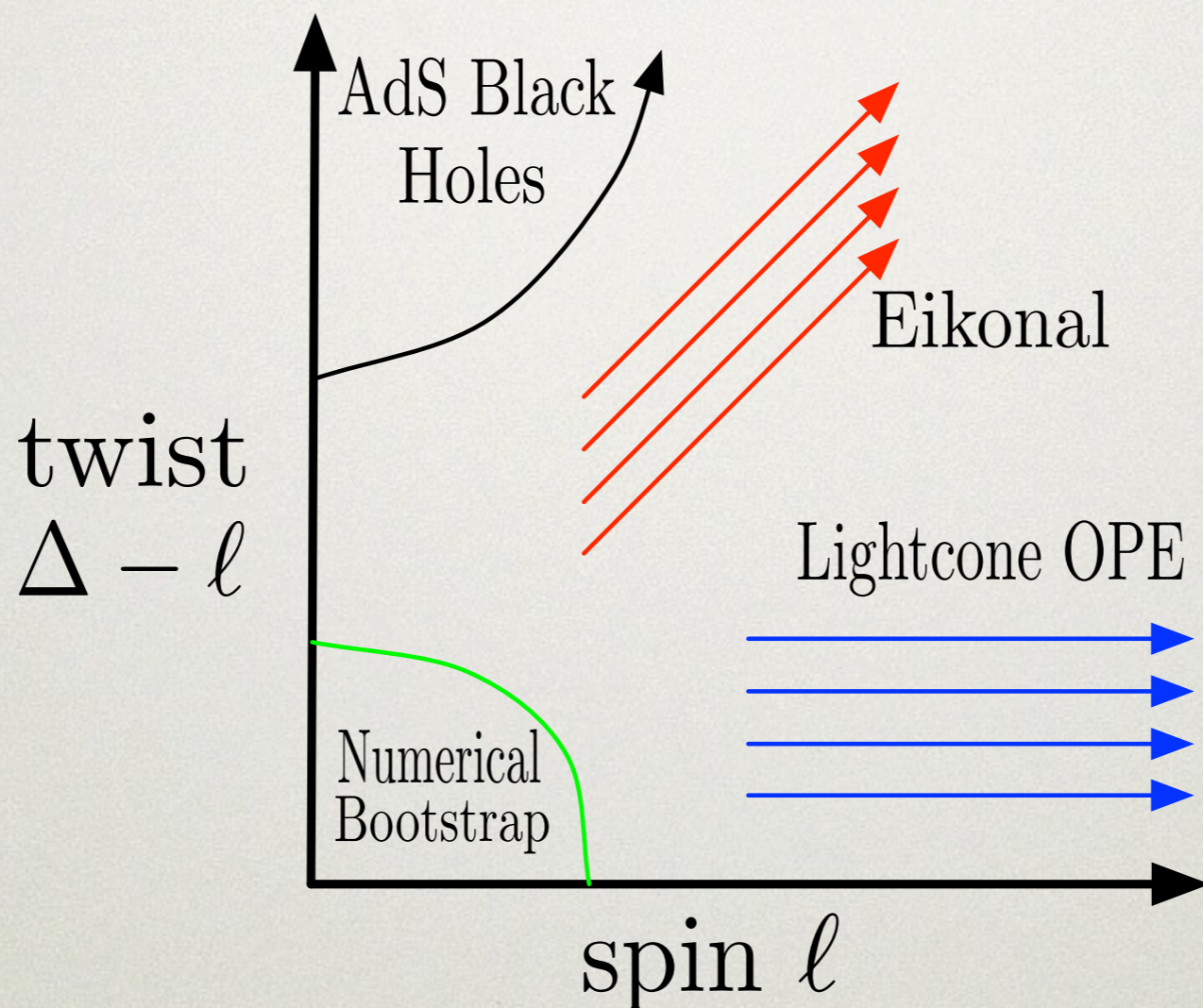
Block 'eikonalizes'. Beyond this limit,
more general series.

**FUTURE
DIRECTIONS
AND
CONCLUSIONS**

CONCLUSIONS

- Long-Range AdS Locality Completely General
- Eikonalization can be seen directly in the bootstrap
- Relation between Eikonalization / Classical Fields and Short-Range locality
- Can begin to improve large spin expansion

FUTURE DIRECTIONS: PHYSICS OF CFT SPECTRA



Need to develop and apply new techniques to understand the full CFT spectrum.

EXTRA SLIDES

WHAT IS THE BOOTSTRAP?

- Conformal Symmetry
- Unitarity
- Crossing Symmetry

What can we learn from the fundamental principles?

The diagram illustrates the crossing symmetry of a four-point correlation function. On the left, a tree-level exchange process is shown where external legs ϕ_1 and ϕ_2 meet at a vertex, and ϕ_3 and ϕ_4 meet at another vertex, with an internal propagator ϕ_k connecting them. The vertex functions are labeled f_{12k} and f_{34k} . A red summation symbol \sum_k is placed to the left of the diagram. On the right, the same process is shown in the crossed channel, where ϕ_1 and ϕ_3 meet at a vertex, and ϕ_2 and ϕ_4 meet at another vertex, with an internal propagator ϕ_k connecting them. The vertex functions are labeled f_{14k} and f_{23k} . A red summation symbol \sum_k is placed to the left of this diagram. An equals sign is placed between the two diagrams.

$$\sum_k \begin{array}{c} \phi_1 \qquad \phi_4 \\ \diagdown \quad \diagup \\ f_{12k} \text{---} \phi_k \text{---} \\ \diagup \quad \diagdown \\ \phi_2 \qquad \phi_3 \\ f_{34k} \end{array} = \sum_k \begin{array}{c} \phi_1 \qquad \phi_4 \\ \diagdown \quad \diagup \\ f_{14k} \text{---} \phi_k \text{---} \\ \diagup \quad \diagdown \\ \phi_2 \qquad \phi_3 \\ f_{23k} \end{array}$$

CONSIDER THE 4-PT CFT CORRELATORS

Recall that 4-pt correlators can be written

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle = \frac{A(u, v)}{(x_{13}^2 x_{24}^2)^{\Delta_\phi}}$$

where the conformal cross-ratios are

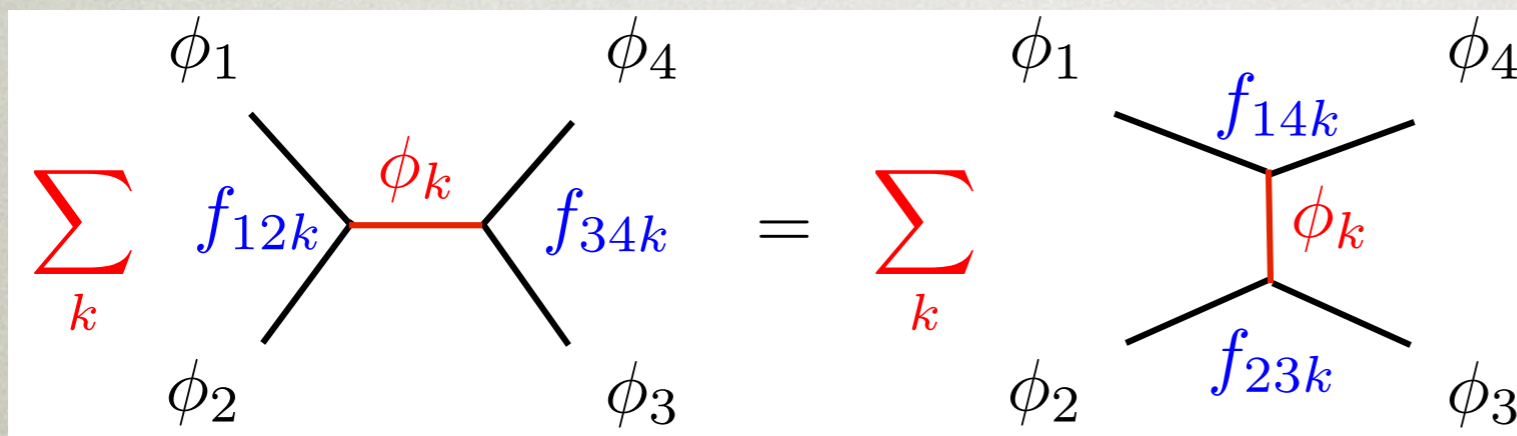
$$u = \left(\frac{x_{12}^2 x_{34}^2}{x_{24}^2 x_{13}^2} \right), \quad v = \left(\frac{x_{14}^2 x_{23}^2}{x_{24}^2 x_{13}^2} \right)$$

We can use elementary quantum mechanics
to rewrite this in a different way...

FORMULATE CFT BOOTSTRAP

Crossing symmetry gives the **Bootstrap Equation**:

$$\frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_\phi}} \sum_{\mathcal{O}} P_{\mathcal{O}} g_{\tau_{\mathcal{O}}, l_{\mathcal{O}}}(u, v) = \frac{1}{(x_{14}^2 x_{23}^2)^{\Delta_\phi}} \sum_{\mathcal{O}} P_{\mathcal{O}} g_{\tau_{\mathcal{O}}, l_{\mathcal{O}}}(v, u)$$



Unitarity:

$$P_{\mathcal{O}} = f^2 > 0$$

WHAT'S SO GREAT ABOUT MELLIN AMPLITUDES?

- Mellin Space is 'Momentum Space for CFTs'
- Use **complex analysis** to study CFTs.
Mellin Amplitudes always **meromorphic**
- Mellin Amplitude **Factorizes** on CFT States
- Algebraic **Feynman Rules** for AdS/CFT
- In Flat Space Limit of AdS/CFT:
Mellin Amplitude becomes S-Matrix, CFT
Bootstrap gives Optical Theorem,
meromorphy becomes analyticity