Locality and Classical Fields in AdS, from the Bootstrap

Jared Kaplan
Johns Hopkins University

based on various work with Liam Fitzpatrick, David Poland, David Simmons-Duffin, Matt Walters. & Junpu Wang

Motivation

Understand the **universal features** of AdS quantum gravity directly from the CFT:

- Fock space? AdS Locality? Long-Range Forces?
- Effective Field Theory description in AdS?
- Classical Field Configurations in AdS?
- Black Hole Microstates - Temperature, Blackness…

Which of these require **further assumptions/restrictions** beyond Conformal Symmetry and QM?

Central CFT Question for This Talk

What can we say about the CFT Spectrum, OPE coefficients, and correlators assuming only that:

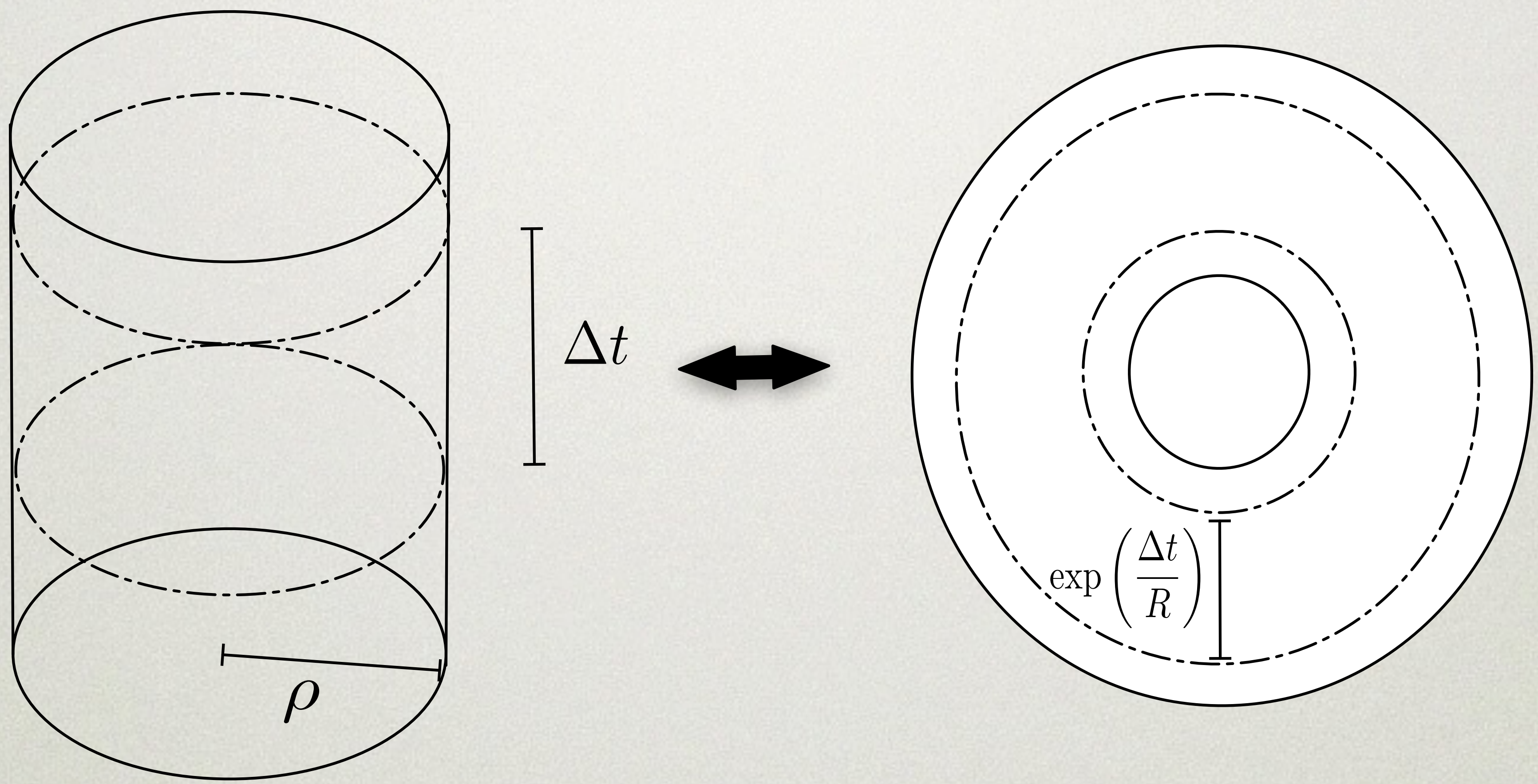
$$\mathcal{O}_1,\ \mathcal{O}_2,\ T \quad \text{with} \quad \mathcal{O}_i(x)\mathcal{O}_i(0) \supset T$$

Answer: Quite a bit, all of it with an AdS Interpretation.

AdS/CFT Kinematics

Energies and Dimensions in AdS/CFT

AdS | CFT



$$H_{AdS} = D_{CFT}$$

Representations of Global Conformal Alg

The momentum and special conformal generators act as raising and lowering operators wrt Dimension

$$[D, P_\mu] = P_\mu \quad [D, K_\mu] = -K_\mu$$

Irreducible reps built from primaries:

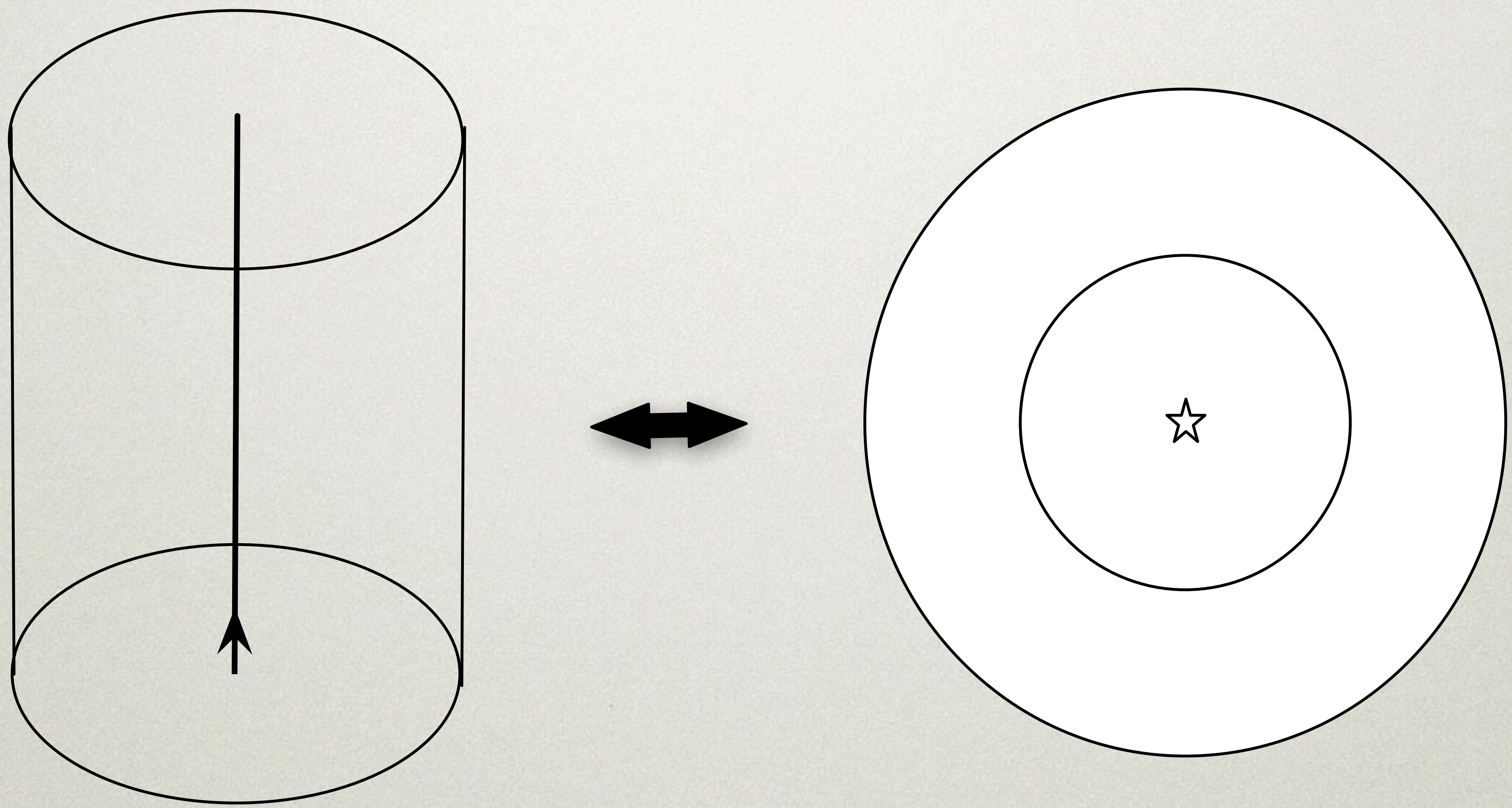
$$[K_\mu, \mathcal{O}(0)] = 0 \;\; \text{or} \;\; K_\mu|\psi_{\mathcal{O}}\rangle = 0$$

Can derive a unitarity relation for $\tau = \Delta - \ell$

$$\Delta_s \geq \frac{d}{2} - 1 \;\; \text{and} \;\; \tau_\ell \geq d - 2$$

Conformal Symmetry and Primary States

AdS | CFT



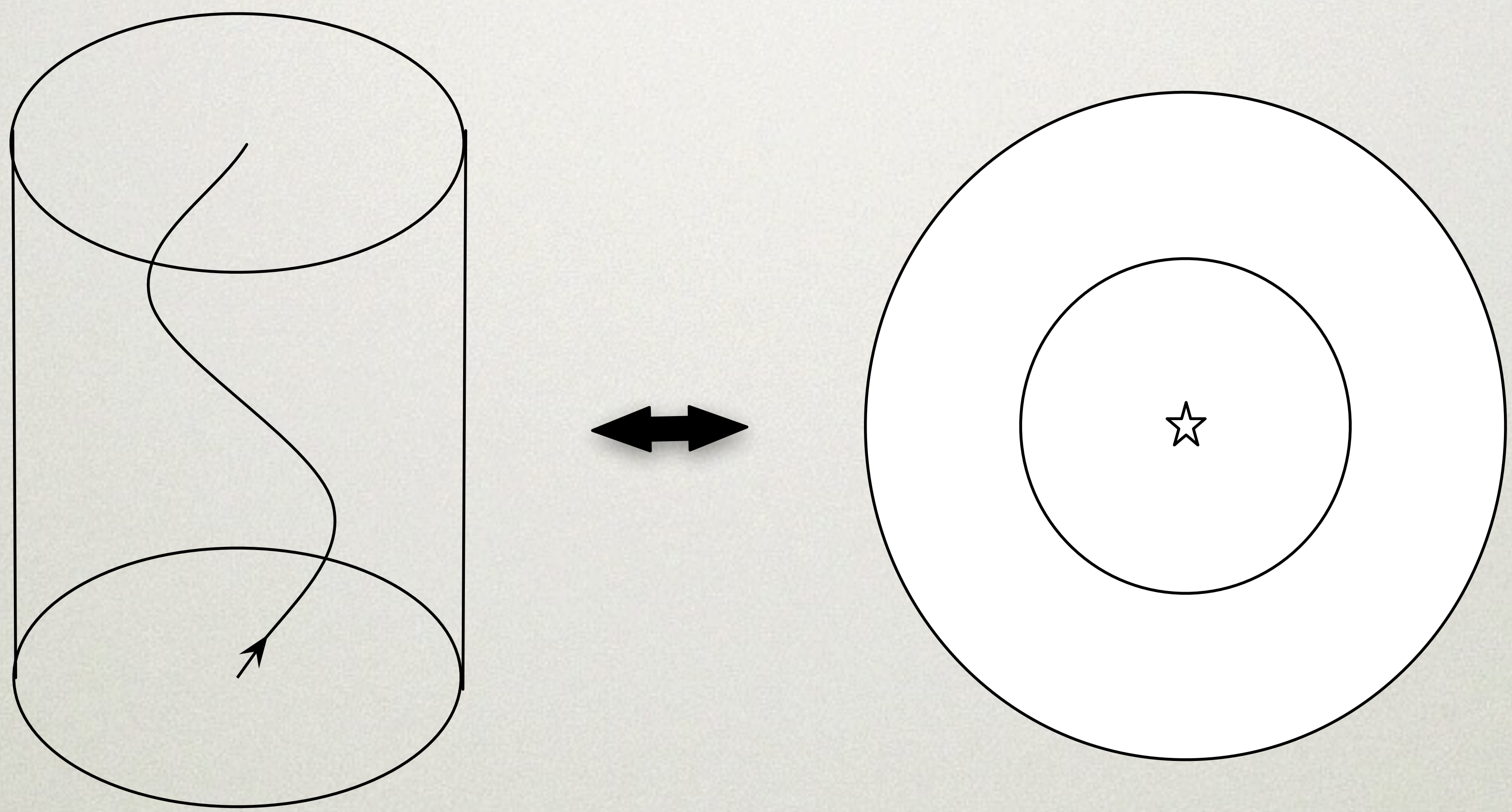
$$\psi_0(t,\rho) = e^{i\Delta t}\cos^{\Delta}(\rho)$$

$$\mathcal{O}|0\rangle$$

CoM of Ground State | CFT Primary State

Excited/Descendant States

AdS | CFT



$\psi_{n,\ell}(t, \rho, \Omega)$

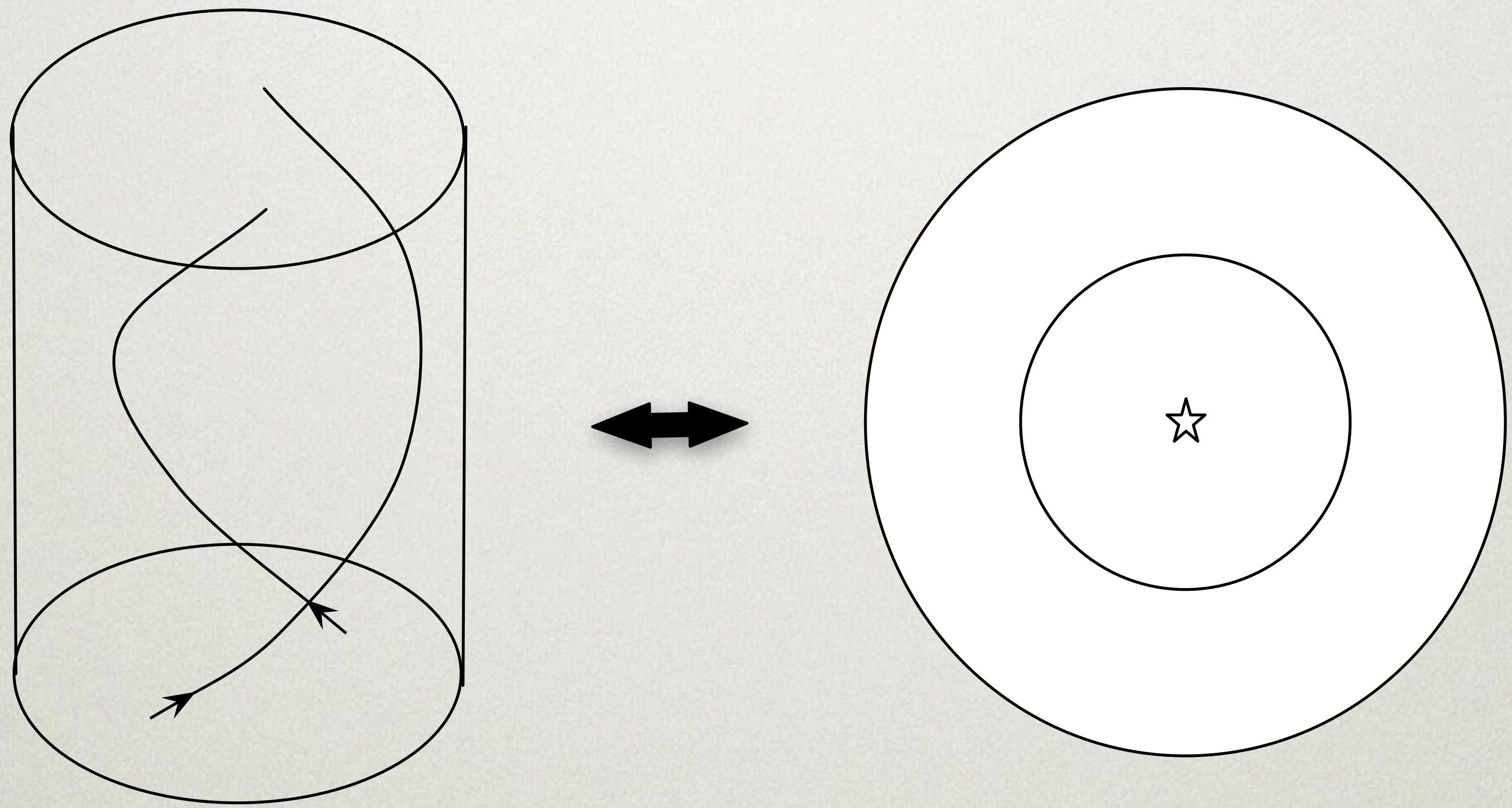
Center of Mass for Excited State

$\left(\partial^{2n} \partial_{\mu_1} \cdots \partial_{\mu_\ell} \mathcal{O}\right) |0\rangle$

Descendant of a Primary

Two Particle States

AdS | CFT



$\psi_{n,\ell}(t_i, \rho_i, \Omega_i)$

$\left(\mathcal{O}\partial^{2n}\partial_{\mu_1}\cdots\partial_{\mu_\ell}\mathcal{O}\right)|0\rangle$

Two Particle State, CoM at Origin

`Double-Trace’ Primary

Long-Distance AdS Locality is Completely Universal

Two Notions of Locality

AdS has an intrinsic length scale: R_{AdS}

$$ds^2 = -\cosh^2\left(\frac{\kappa}{R_{AdS}}\right)dt^2 + d\kappa^2 + \sinh^2\left(\frac{\kappa}{R_{AdS}}\right)d\Omega^2$$

Long-Distance Locality: Particles/Systems decouple when separated by $D \gg R_{AdS}$

Micro-Locality: There exists a good Effective Field Theory Description in AdS with $\Lambda_{EFT} \gg \frac{1}{R_{AdS}}$

Formal Definition of Long-Distance Locality?

$\psi_c =$ 

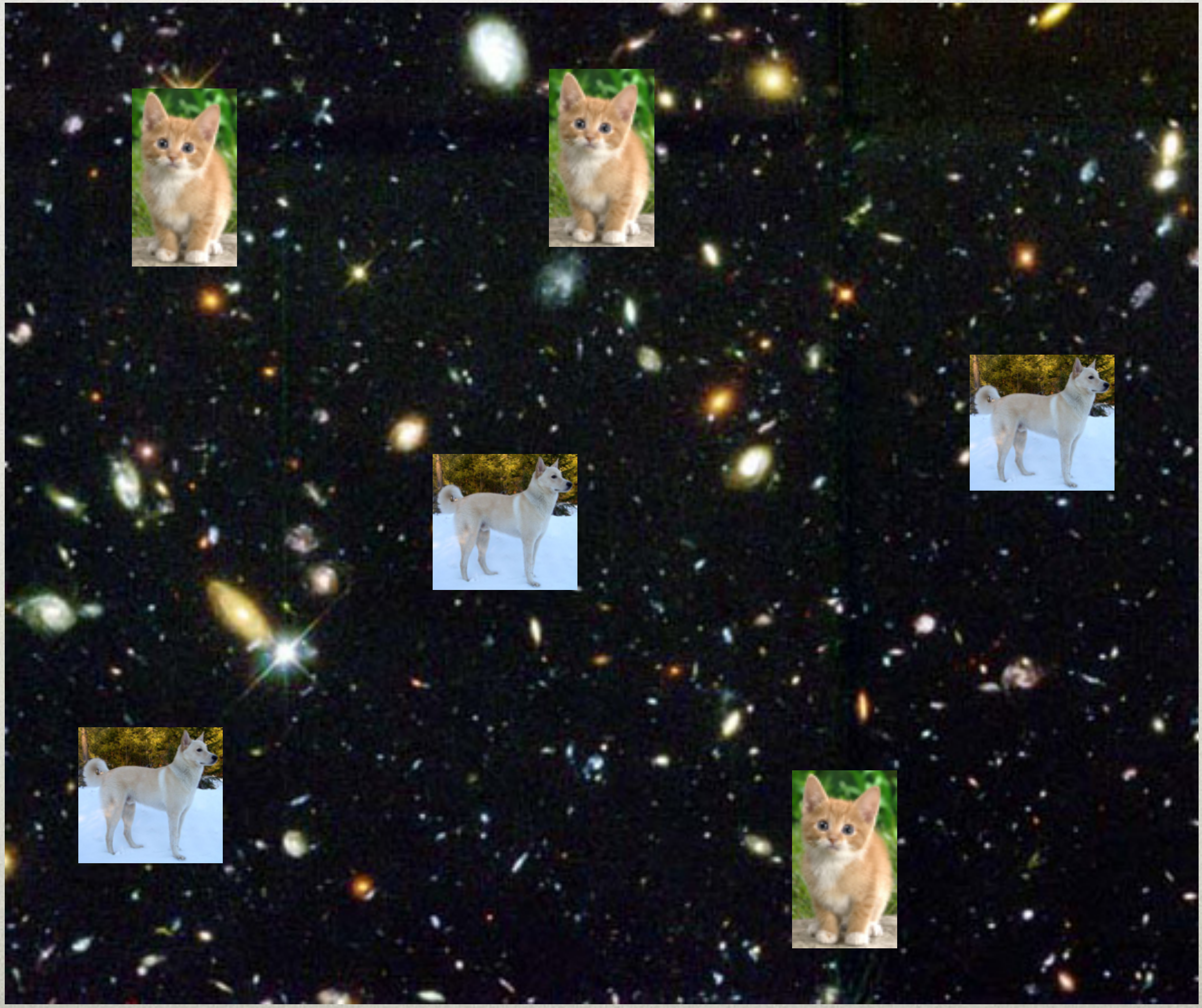
$\psi_d =$ 

$\implies \exists\ \psi_{cd} =$ 

Statement about **structure of the Hilbert Space**:

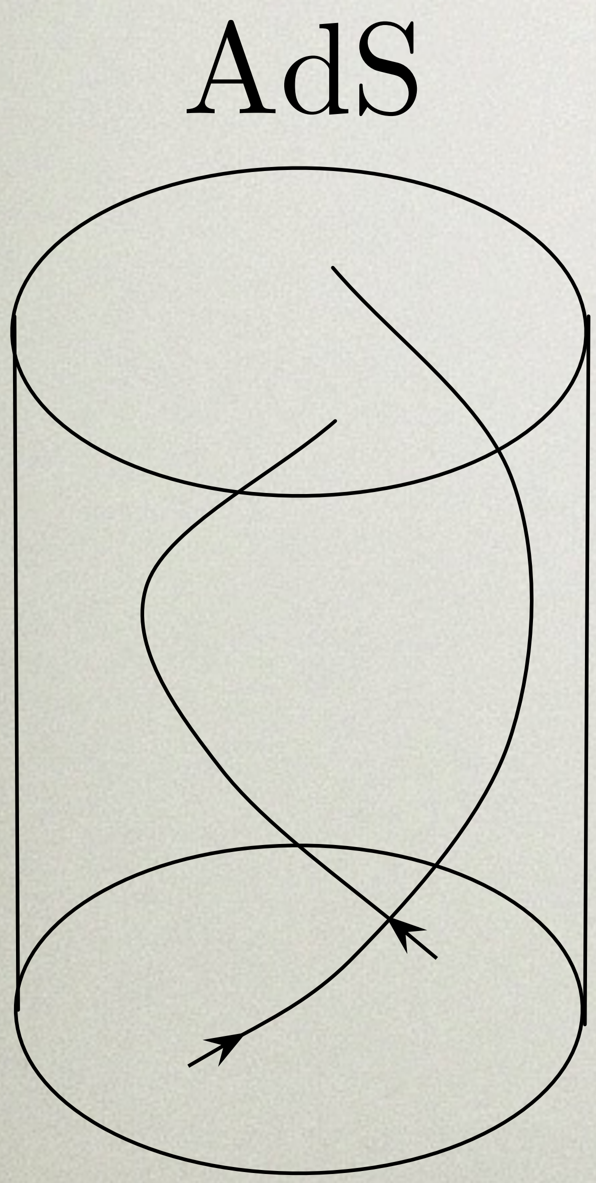
$$\mathcal{H}_{AdS} = \mathcal{H}_{CFT} \approx \text{Fock Space}$$

A Fock Space at Large Separation



How to Define Distant Furballs?

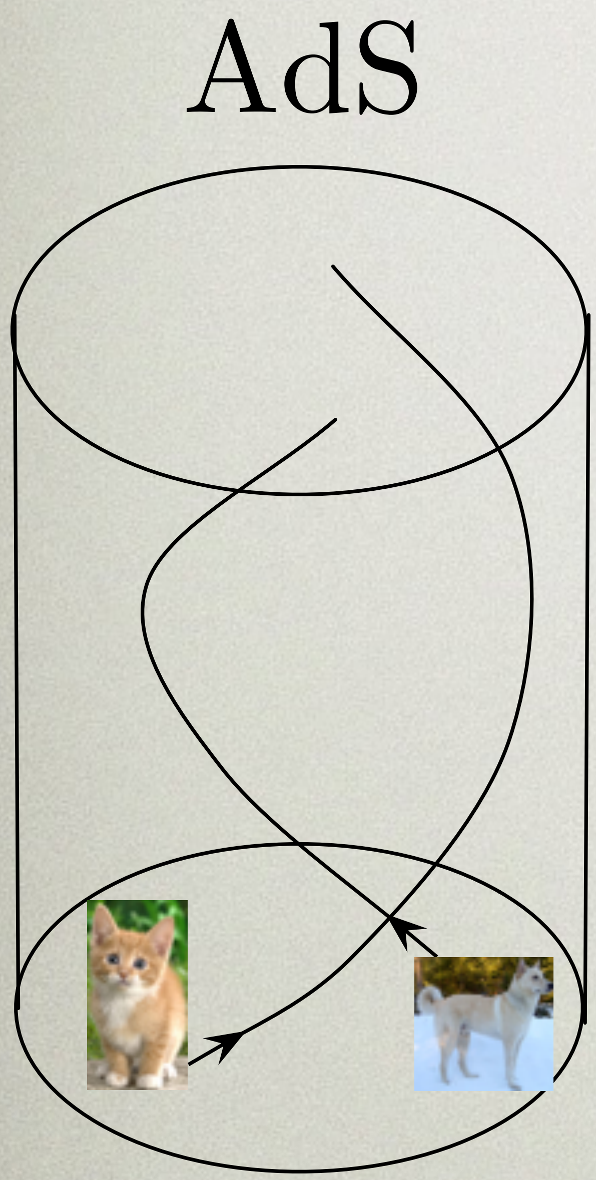
Geodesic separation between cat & dog:



$$\kappa \sim R_{AdS} \log \ell$$

Are there two furball states at **large angular momentum**???

Two Furball Physics?



AdS Energy = CFT Dimension:

$$E_{n\ell} = E_c + E_d + 2n + \ell + \gamma(n, \ell)$$

Anomalous dimension, $\gamma(n, \ell)$, is a kind of **`binding energy'**.

Existence of states as $\ell \to \infty$ with vanishing $\gamma(n, \ell)$ implies **AdS Cluster Decomposition**.

General Theorem (Any CFT in d>2)

Consider OPE of **any** two scalar primary operators:

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{\Delta,\ell} c^{12}_{\Delta,\ell}\mathcal{O}_{\Delta,\ell}(0)$$

For each n there **exists** a tower of operators

$$\mathcal{O}_{\Delta,\ell} \quad \text{with} \quad \Delta = \Delta_1 + \Delta_2 + 2n + \ell + \gamma(n,\ell)$$

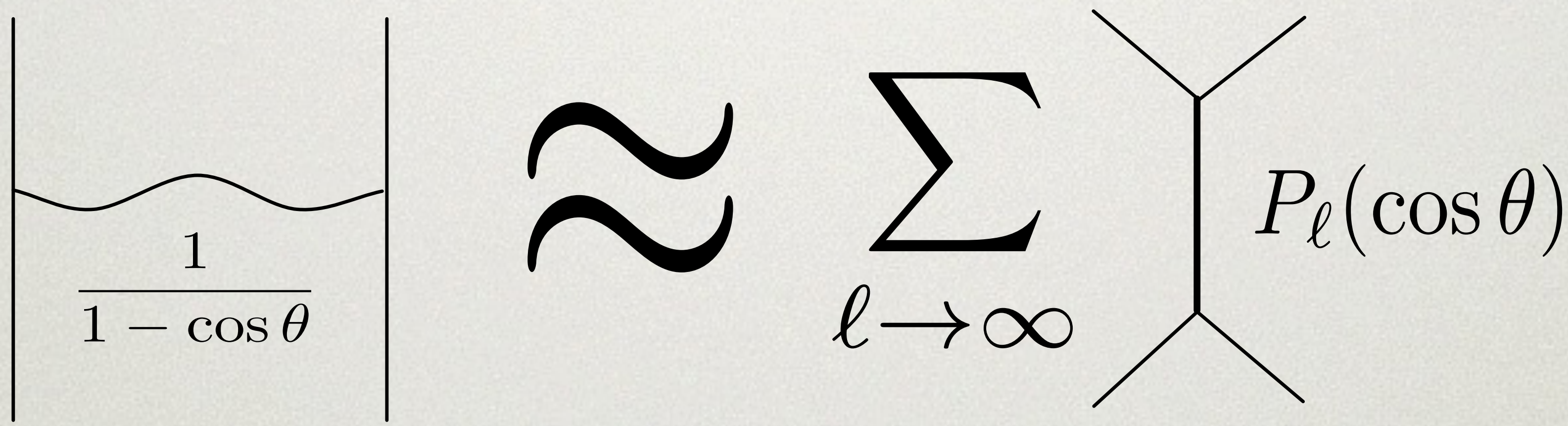
as $\ell \to \infty$, where the anomalous dimensions

$$\gamma(n,\ell) \to \frac{\gamma_n}{\ell^{\tau_m}} \quad \text{or} \quad \gamma_n e^{-\tau_m \kappa}$$

from leading twist exchange, generically $T_{\mu\nu}$

The Idea of the Proof: A Scattering Analogy

Free propagation and massless exchange require large amplitude at large ℓ, e.g.



Completely analogous CFT phenomenon. Implies existence of large ℓ states.

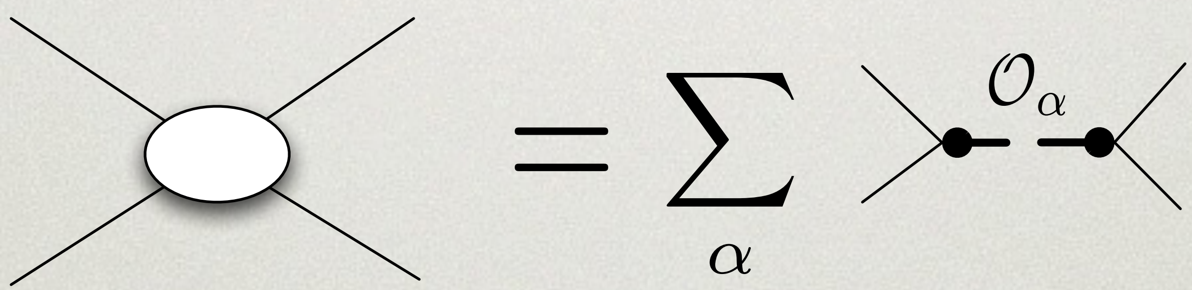
Partial Wave Amplitudes -> Conformal Partial Waves

The Conformal Partial Wave Decomposition

Insert 1, organize according to conformal symmetry:

$$\langle \mathcal{O}_1 \mathcal{O}_2 \left(\sum_\alpha |\alpha\rangle\langle\alpha| \right) \mathcal{O}_3 \mathcal{O}_4 \rangle$$

Since **operators = states** in the CFT, write 4-pt as



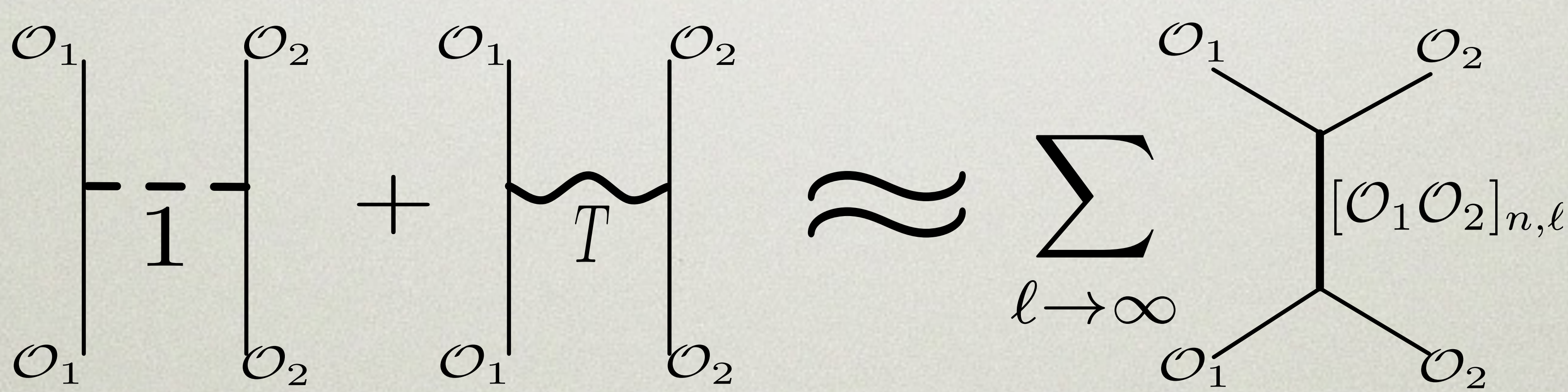
A sum over exchange labeled by **primary operators**, magnitude given by product of 3-pt correlators.

Light-Cone OPE Limit (Diagrams = Blocks)

The equivalent of a t-channel singularity in the CFT is the light-cone OPE limit:

$$\langle \mathcal{O}_1 \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_2 \rangle = u^{-\frac{\Delta_1+\Delta_2}{2}} \left(1 + u^{\frac{\tau_T}{2}} f_T(v) + \cdots \right)$$

giving a bootstrap equation



Example: long-range gravity is completely universal.

Micro-Locality for Special CFTs

Micro-Locality (or flat space locality)

Showed that **All CFTs** have a Fock Space spectrum at long-distances. Would need at **short-distances** too.

To make progress, need **perturbative** expansion. (Our definition is in terms of AdS EFT.)

Both are **special** features, invalid for most CFTs.

Micro-Locality (or flat space locality)

Need a 3rd special feature. Counter-example:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{N^2}\left[\lambda\phi^4 + R_{AdS}^4(\partial\phi)^4 + \cdots\right]$$

To see what went wrong, look at flat space S-Matrix:

$$\mathcal{A} = \frac{1}{N^2}\left(\lambda + R^4(s^2+t^2+u^2) + R^8 s^4 + R^{12} S^6 + \cdots\right)$$

Breaks down at the scale R, despite weak coupling.

EFTs well-approximated by polynomial Amplitudes.

Mellin Space as AdS Momentum Space

Can define Mellin Amplitude for CFT correlator:

$$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_4) \rangle \propto \int \frac{ds dt}{(2\pi i)^2} \tilde{M}(s,t) u^{-s} v^{-t}$$

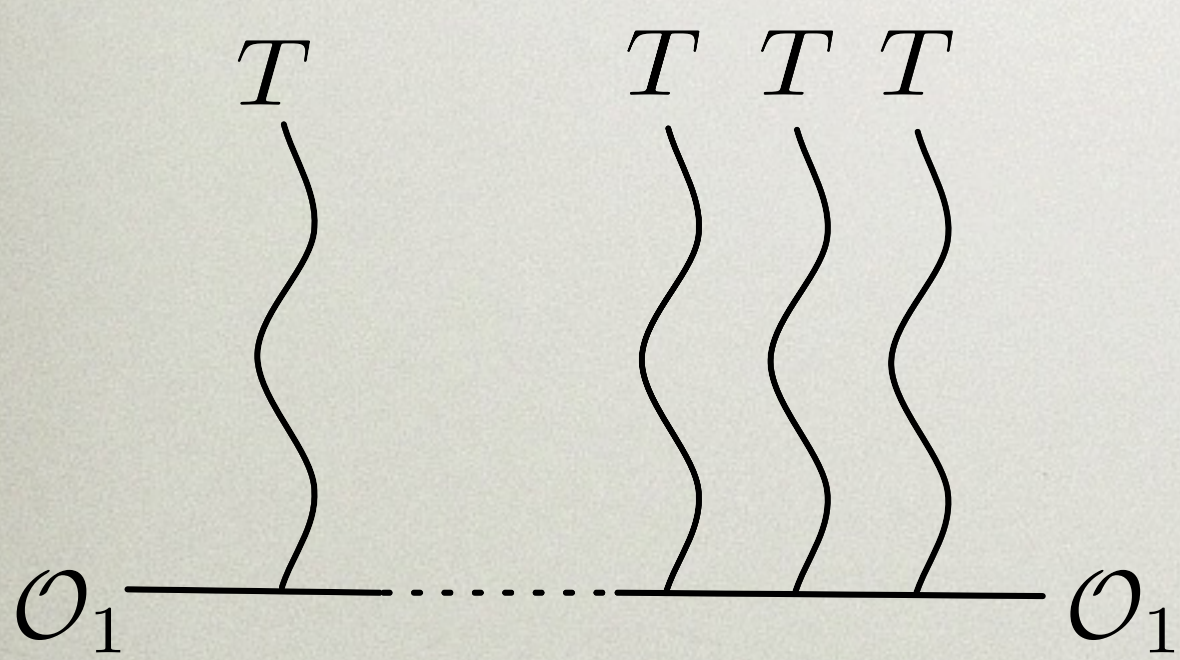
Mellin variables direct analog of Mandelstam s and t.

Mellin Amplitude is `momentum space' for AdS.
Asymptotic bounds —> EFT description in AdS.

Classical AdS Fields and `Eikonalization'

Multiple Exchanges and Classical Fields

In any spacetime, effective classical fields emerge from the summation of infinite series of simple diagrams, such as:



$$\Longrightarrow V(r) = \frac{GM}{r^{d-2}}$$

Can get Schwarzschild by including interactions.

Inter-related Questions:

Does the Bootstrap know about the multiple exchanges that build up a classical field in AdS?

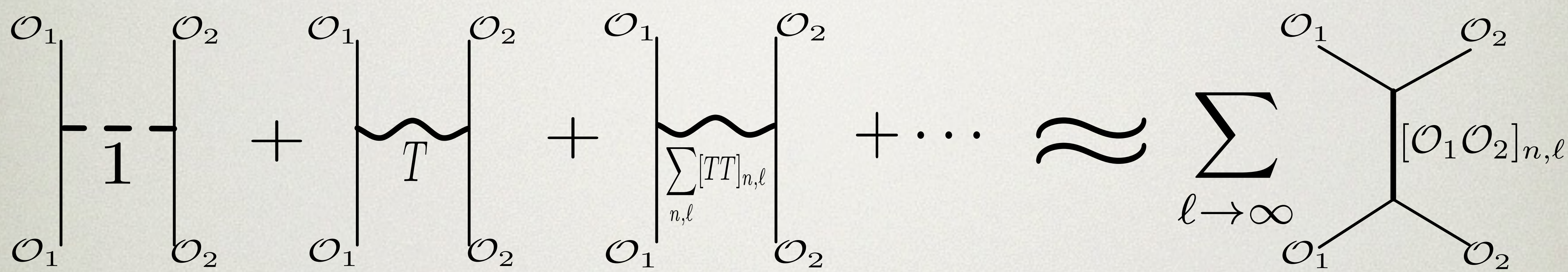
Why are Mellin amplitude asymptotics important?

Can we take $\frac{1}{\ell}$ perturbation theory further?

How far can we go with no assumption except:

$$\mathcal{O}_1,\ \mathcal{O}_2,\ T \quad \text{with} \quad \mathcal{O}_i(x)\mathcal{O}_i(0) \supset T$$

A Bootstrap Implication



On the right-hand side, $[\mathcal{O}_1\mathcal{O}_2]_{n,\ell}$ anomalous dim:

$$g_{\tau,\ell}(v,u) \approx \left(1 + \frac{\gamma_n}{2\ell^{\tau_m}}\ln v + \frac{\gamma_n^2}{8\ell^{2\tau_m}}\ln^2 v + \cdots\right) g_{\tau_n,\ell}(v,u)$$

How can we match this on the left-hand side?

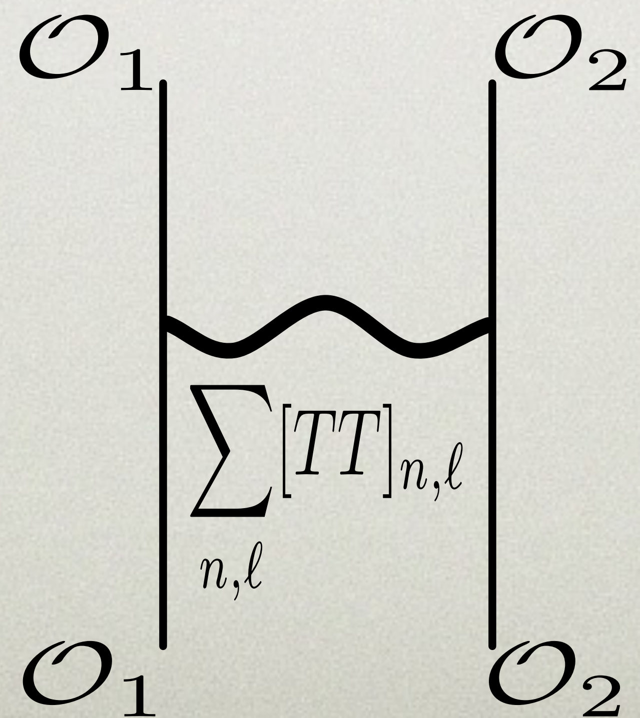
Not obvious because Bootstrap is `on-shell'.

Be Fruitful and Multiply...

Fock Space Theorem implies existence of:

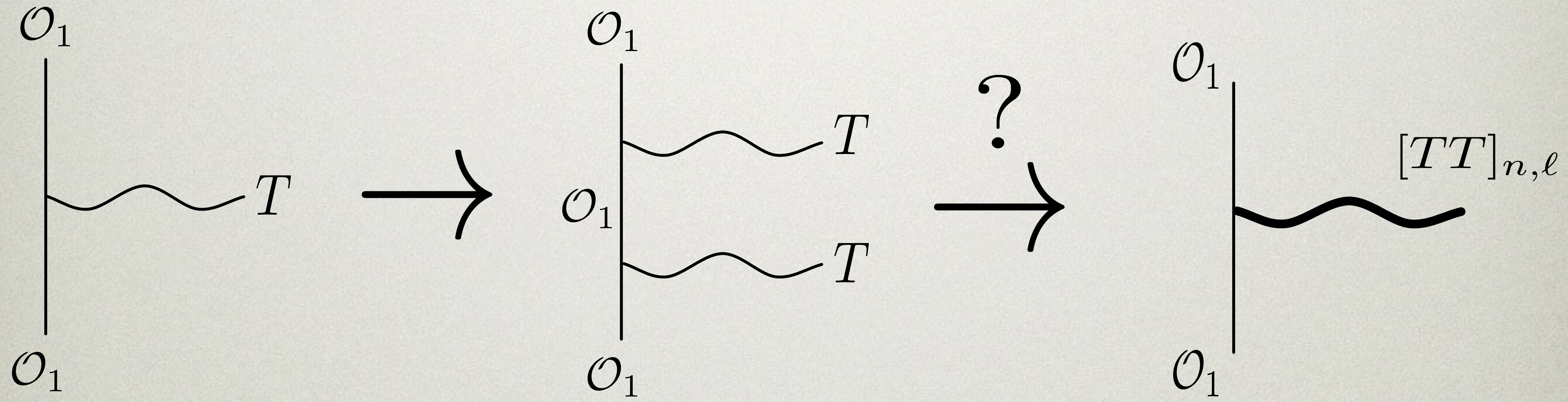
$$[\mathcal{O}_1\mathcal{O}_2]_{n,\ell},\ [\mathcal{O}_1 T]_{n,\ell},\ [TT]_{n,\ell},\ \cdots,\ [[\mathcal{O}_1\mathcal{O}_1]_{n,\ell}T]_{n',\ell'},\cdots$$

So **at large spin**, we are allowed to ask about:



A Needed OPE Limit

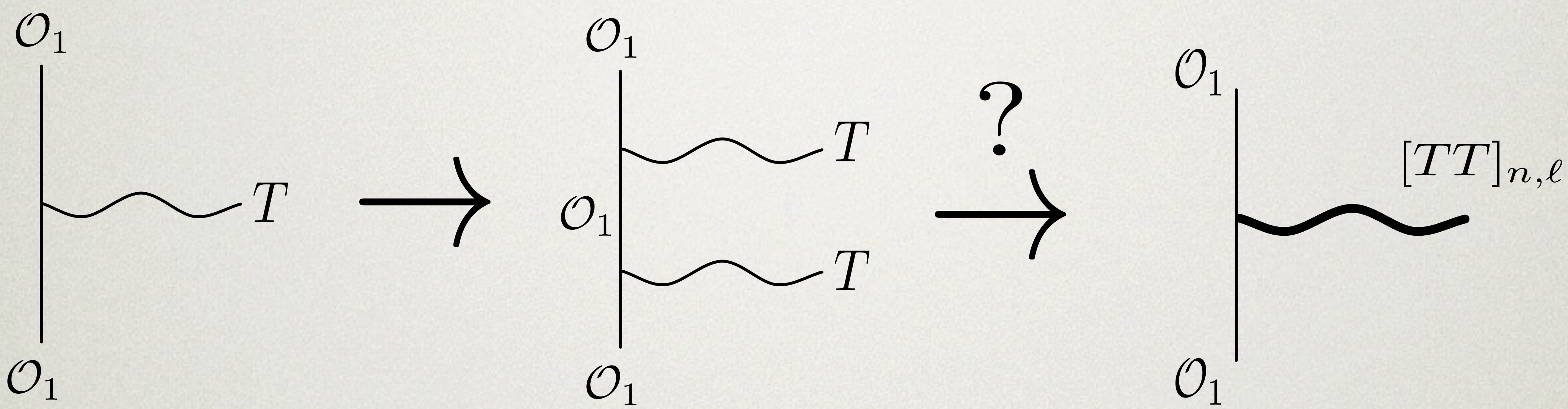
We would like to use the OPE to deduce at large spin:



Without further assumptions, ill-defined!!

Cross-channel OPE limits of Blocks do not exist.

A Needed OPE Limit



Need to expand a function such as

$$ {}_2F_1(1,1,2,z) = -\frac{\log(1-z)}{z} $$

Near $z = 1$

One reason why crossing symmetry so non-trivial!

Needed OPE Limit, Understood from Mellin

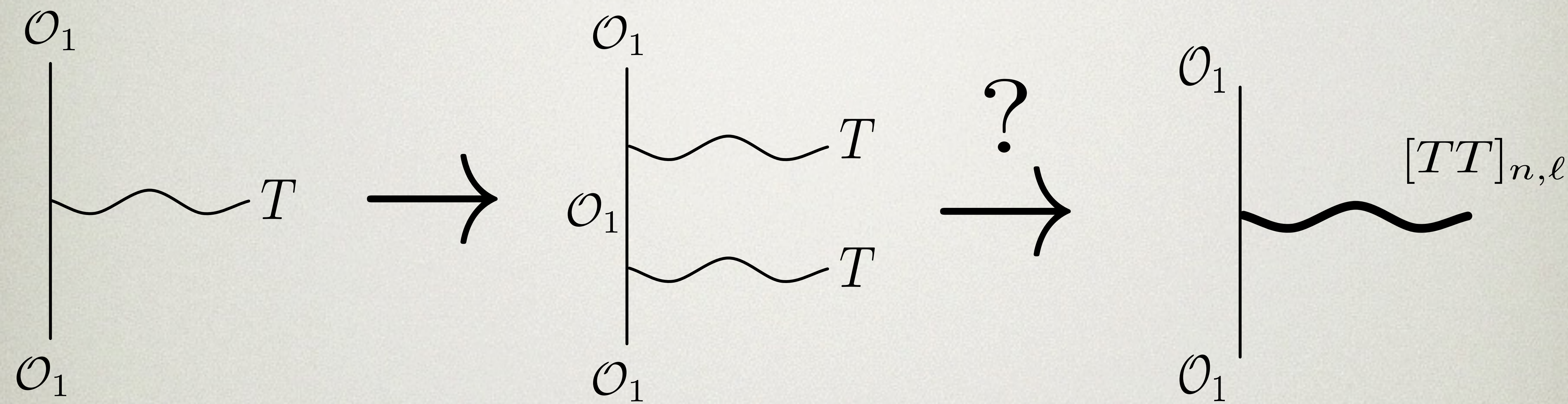
Toy example: $$G(z) = \int_{-i\infty}^{i\infty} M(\delta)\, z^{-\delta} d\delta$$

Both OPE limits, $z = 0$ and $z = 1$ can be **studied simultaneously.**

Behavior near $z = 0$ **determined by poles.**

Behavior near $z = 1$ **determined by asymptotics.**

A Needed OPE Limit, Understood via Mellin

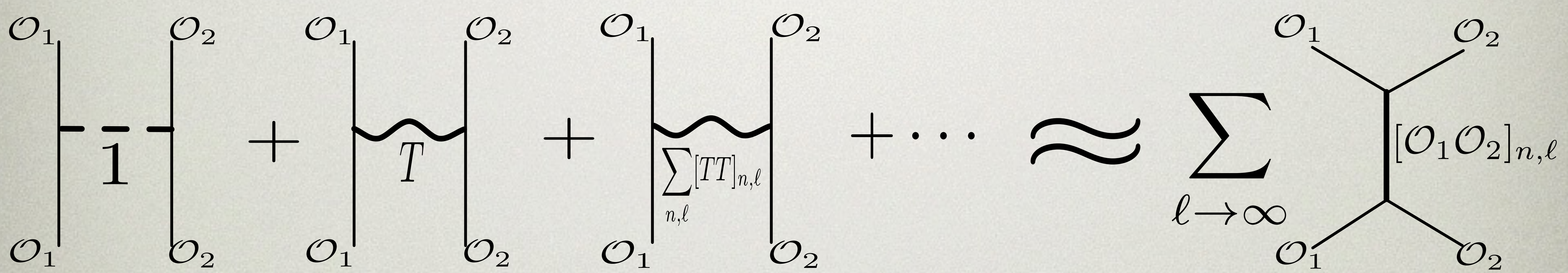


Pathology due to bad asymptotic behavior.

Result controlled by leading Mellin pole, which gives the **universal behavior** expected from the bootstrap equation.

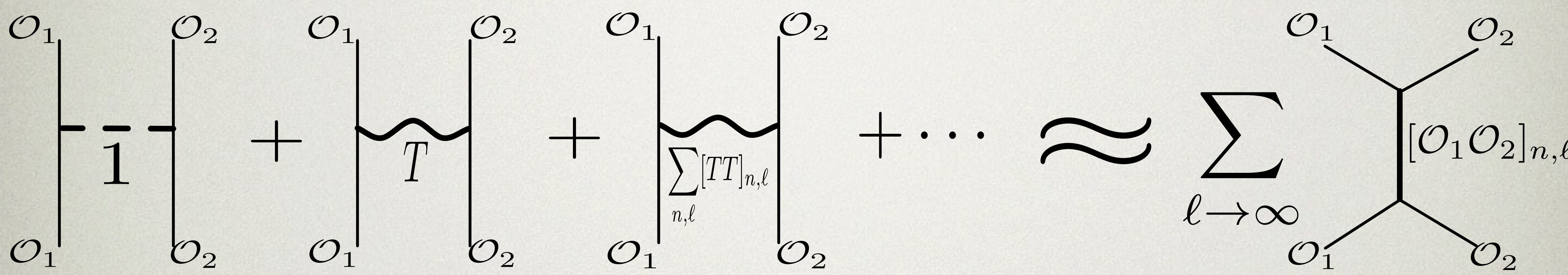
For any CFT, Conformal Blocks can be Summed

If Mellin Amplitude is exponentially bounded, we can sum or `Eikonalize' the conformal blocks:



AdS QFT aka 1/N Perturbation Theory is a special case.

For any CFT, Conformal Blocks can be Summed



In the limit $\quad \tau_T \ll \Delta_1, \Delta_2$

$$e^{c_{11}^T c_{22}^T g_T(u,v)}$$

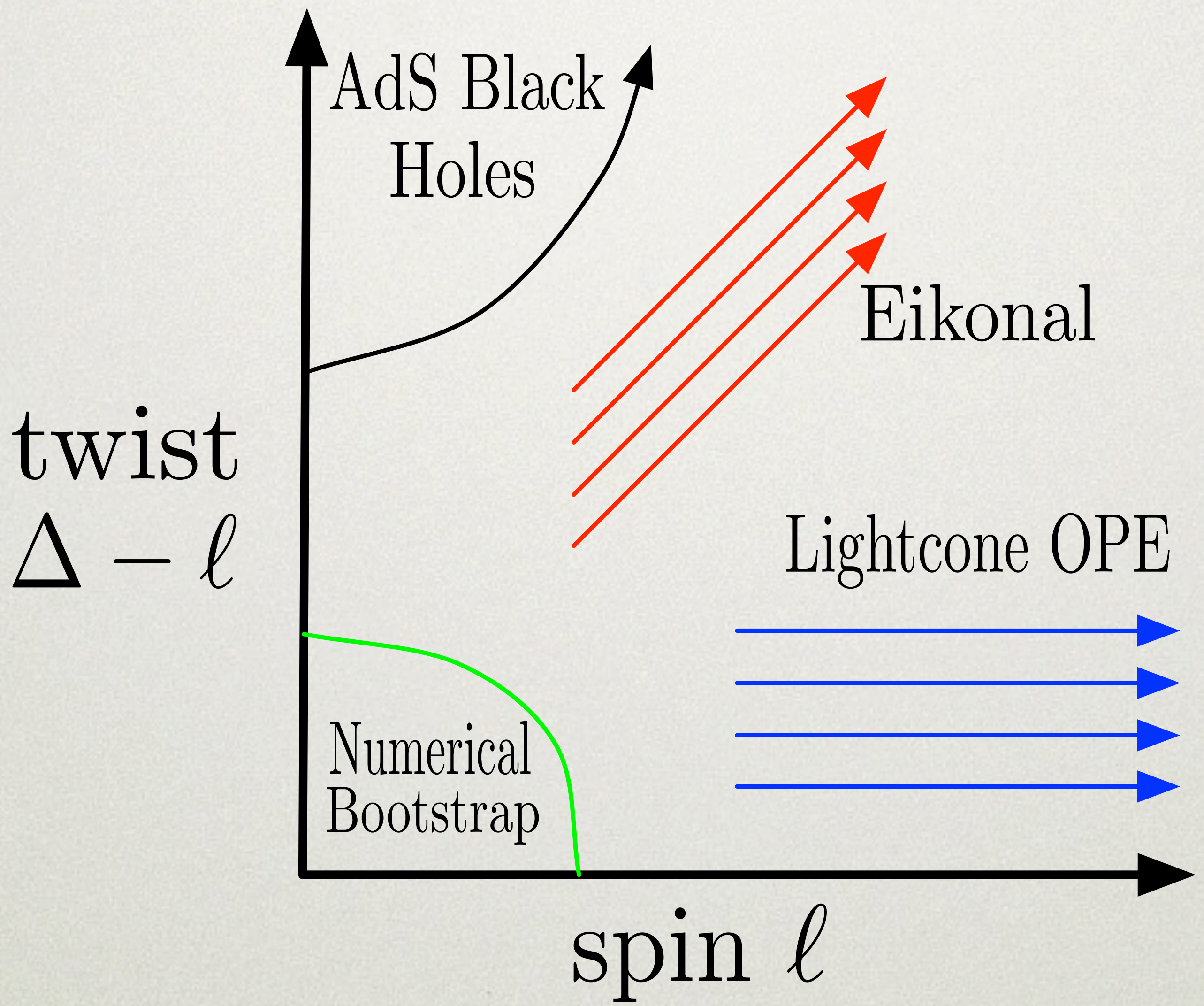
Block `eikonalizes'. Beyond this limit, more general series.

Future Directions and Conclusions

Conclusions

- Long-Range AdS Locality Completely General
- Eikonalization can be seen directly in the bootstrap
- Relation between Eikonalization/Classical Fields and Short-Range locality
- Can begin to improve large spin expansion

Future Directions: Physics of CFT Spectra



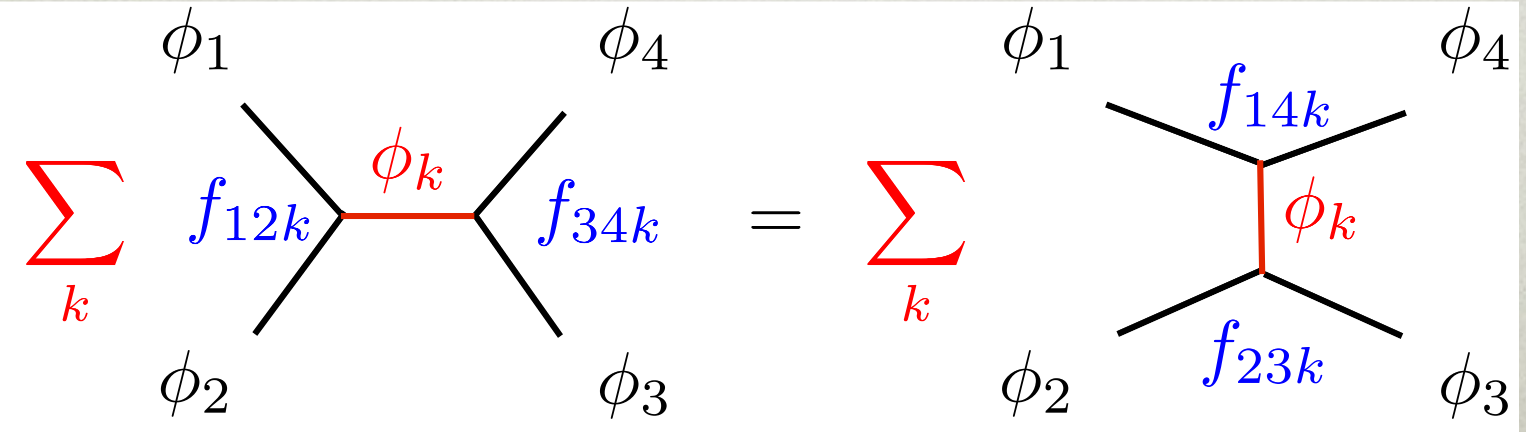
Need to develop and apply new techniques to understand the full CFT spectrum.

Extra Slides

What is the Bootstrap?

- Conformal Symmetry
- Unitarity
- Crossing Symmetry

What can we learn from the fundamental principles?



Consider the 4-pt CFT Correlators

Recall that 4-pt correlators can be written

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{A(u,v)}{(x_{13}^2 x_{24}^2)^{\Delta_\phi}}$$

where the conformal cross-ratios are

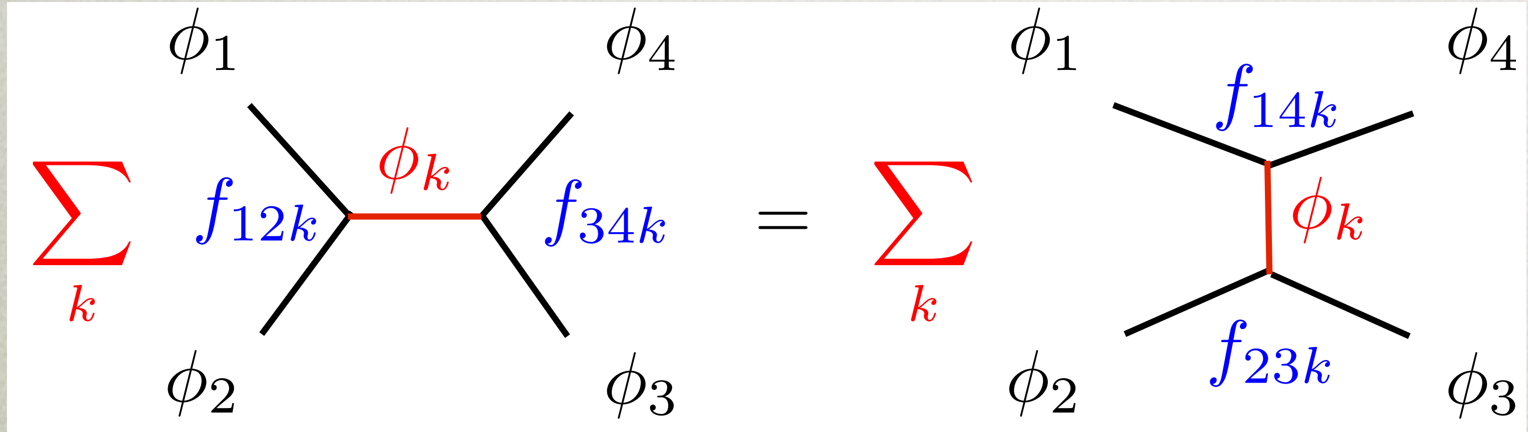
$$u = \left(\frac{x_{12}^2 x_{34}^2}{x_{24}^2 x_{13}^2}\right), \qquad v = \left(\frac{x_{14}^2 x_{23}^2}{x_{24}^2 x_{13}^2}\right)$$

We can use elementary quantum mechanics to rewrite this in a different way...

Formulate CFT Bootstrap

Crossing symmetry gives the **Bootstrap Equation**:

$$\frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_\phi}} \sum_{\mathcal{O}} P_{\mathcal{O}} g_{\tau_{\mathcal{O}},\ell_{\mathcal{O}}}(u,v) = \frac{1}{(x_{14}^2 x_{23}^2)^{\Delta_\phi}} \sum_{\mathcal{O}} P_{\mathcal{O}} g_{\tau_{\mathcal{O}},\ell_{\mathcal{O}}}(v,u)$$



Unitarity:

$$P_{\mathcal{O}} = f^2 > 0$$

What's so great about Mellin Amplitudes?

- Mellin Space is `Momentum Space for CFTs'
- Use **complex analysis** to study CFTs. Mellin Amplitudes always **meromorphic**
- Mellin Amplitude **Factorizes** on CFT States
- Algebraic **Feynman Rules** for AdS/CFT
- In Flat Space Limit of AdS/CFT: Mellin Amplitude becomes S-Matrix, CFT Bootstrap gives Optical Theorem, meromorphy becomes analyticity