On instabilities of non-Fermi liquids

Gonzalo Torroba
Centro Atómico Bariloche,
Argentina

- "Aspects of renormalization in finite density field theory"
L. Fitzpatrick, G. Torroba, H. Wang [arXiv:1410.6811]

- "Enhanced pairing in quantum critical metals near d=3+1"
L. Fitzpatrick, S. Kachru, J. Kaplan, S. Raghu, G. Torroba, H. Wang [arXiv:1410.6814]

→ • "On instabilities of non-Fermi liquids"
S. Raghu, G. Torroba, H. Wang [to appear]

Aspen QFT Conference, February 2015

Systems with a large number of degrees of freedom often display surprising *emergent phenomena*

criticality, dual descriptions, confinement, superconductivity, ...

General strategy: develop theories of matter by looking for universal dynamics at long distance

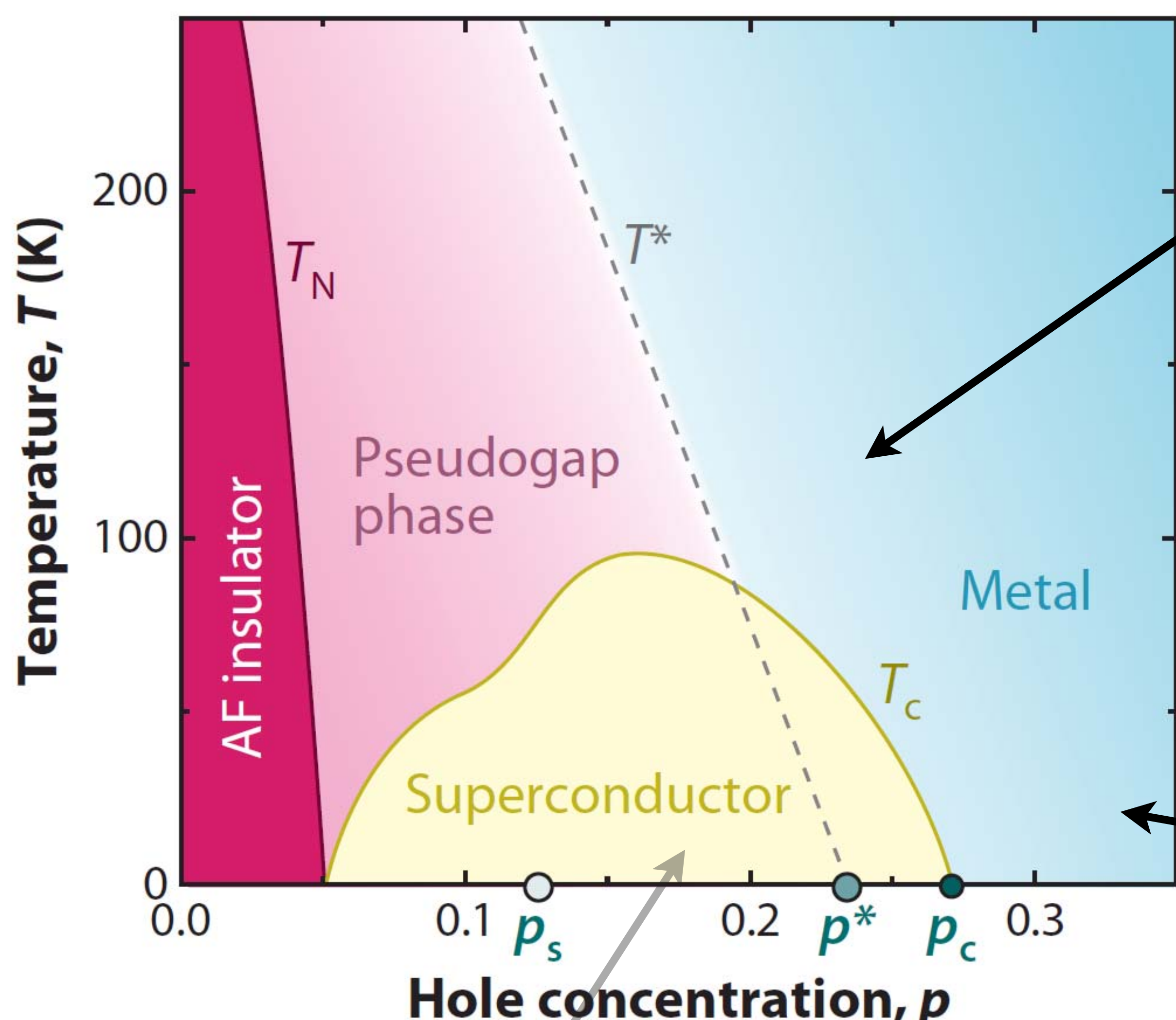
➡ Most developed: *relativistic QFT.*
E.g. phases of gauge theories in 3+1 dimensions include free phases, CFTs, Higgs, confinement

Well understood in certain \`toy' models, such as perturbative theories, large N, SUSY

... still far from "realistic" gauge theories (QCD)

QFT at finite density arises as low energy limit of various condensed matter and high energy systems.
However, much less developed than relativistic case!

Understanding finite density QFT is urgently needed, given spectacular discoveries in strongly correlated electronic systems



Non-Fermi liquid

Fermi liquid

quantum critical point (?)

Can we understand universal properties of these systems using low energy QFT?

➡ Our strategy: develop finite density QFT in perturbative regime (then try to extend to strong coupling)

Objective: determine low energy dynamics in regime where corrections to quasiparticle picture are present

$$G_F(p) = \frac{Z^{-1}(p)}{i\omega - Z_v(p)\varepsilon_k}$$

quasiparticle residue ← $Z^{-1}(p)$

velocity renormalization ← $Z_v(p)$

- NFL effects on superconductivity (SC)?
- quantum critical points (QCP)
- transition between SC and QCP?

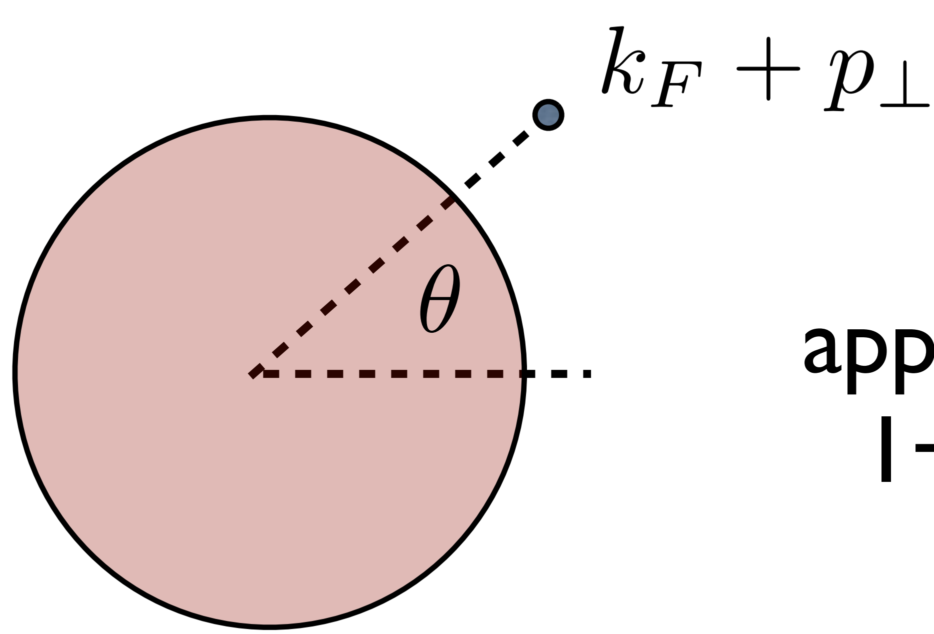
A. Basics of finite density QFT

Electrons in a metal with strong Coulomb interactions are mapped to EFT of weakly coupled quasiparticles:

$$S = \int dt\, d^3k\, \psi^\dagger(k)\big(i\partial_t - (\varepsilon(k) - \mu_F)\big)\psi(k)$$

quasiparticle energy $\qquad \varepsilon(k) = \dfrac{\vec{k}^2}{2m}$

Fermi surface locality:



approx. decoupled
1+1D fermions

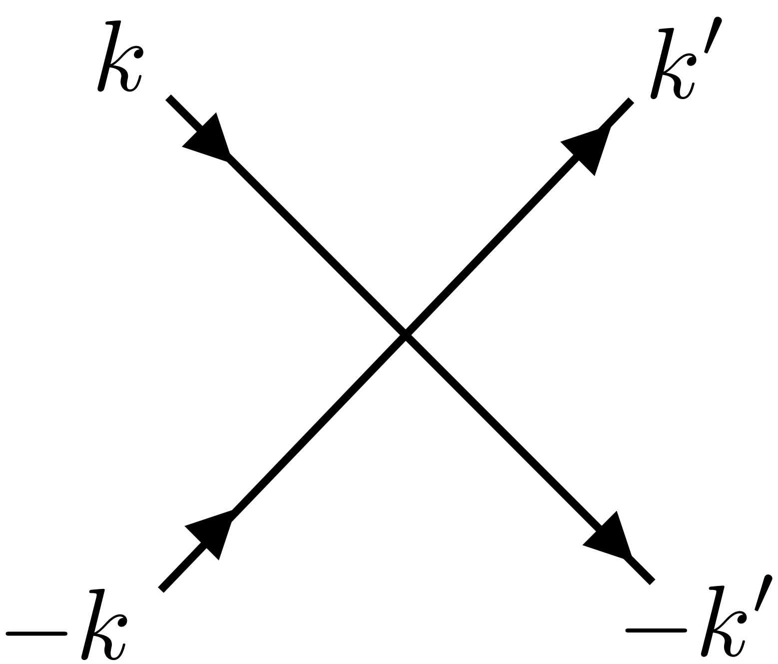
$$\psi_\theta(p_\perp)$$

Excitations and pairing mechanisms

$$\delta n_k = \psi^\dagger(k)\psi(k)$$ density fluctuations (shape of FS)

$$\Psi = \psi(k)\psi(-k)$$ Cooper operator, U(1) phase fluctuation

Pairing by (classically) marginal 4-F interaction:

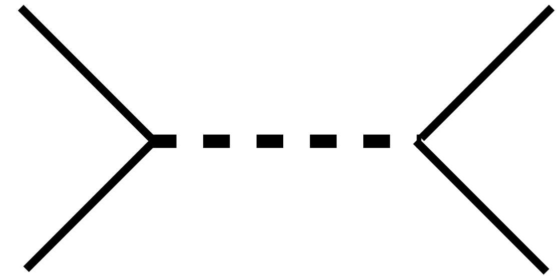


$$L_{int} = V(k-k')\psi(k)\psi(-k)\,\psi^\dagger(k')\psi^\dagger(-k')$$

$$V_{BCS} = const \qquad V_{phonon}(q) = \frac{\omega_q^2}{q_0^2 + \omega_q^2}$$

Stronger pairing in high Tc materials

e.g. $V(q) = \frac{1}{q_0^2 + \vec{q}^{\,2}}$



emergent scalar

Also, QCD at finite density

The model

Fermi surface coupled to massless scalar in $d = 3 - \epsilon$

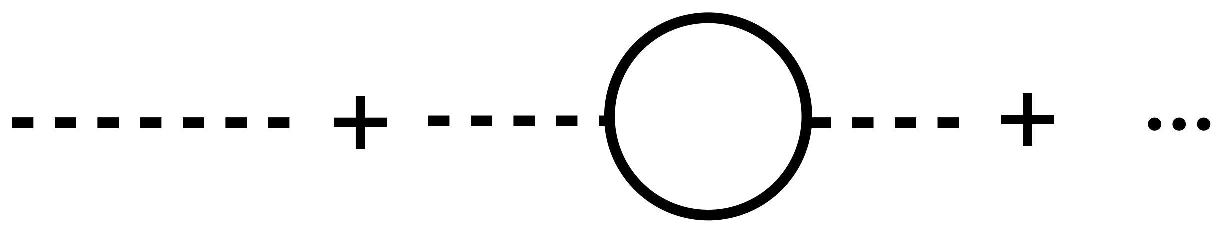
SU(N) global symmetry

$$L_{int} = v^{1/2} \frac{g}{\sqrt{N}} \phi_{ij} \psi_i^\dagger \psi_j + \frac{v}{k_F^2} \frac{\lambda}{N} \psi_i^\dagger \psi_j^\dagger \psi_j \psi_i$$

Will combine → perturbative expansion

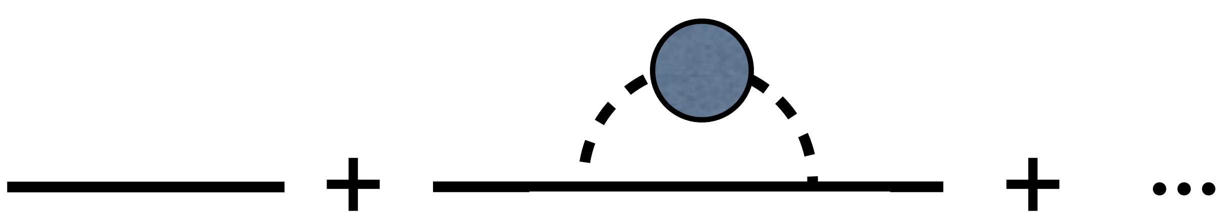
→ large N

Quantum effects



$$D(q) = \frac{1}{q_0^2 + q^2 + M_D^2 \, |q_0/q|}$$

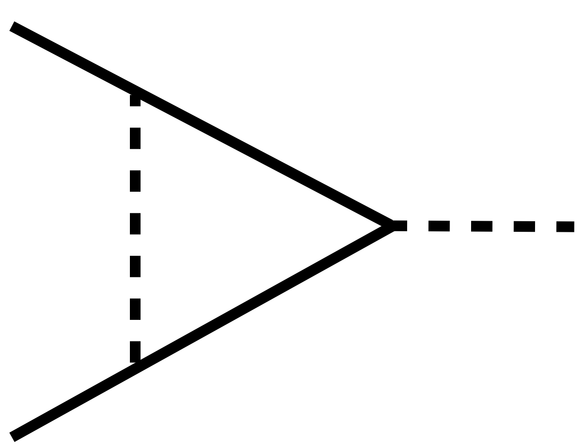
dynamical exponent $z = 3$



$$G(p) = \frac{1}{iZ(\omega)\omega - v_0 p_\perp}$$

$$Z(\omega) = \omega^{-2\gamma} \qquad \gamma = \frac{g^2}{24\pi^2}$$

independent of momentum!

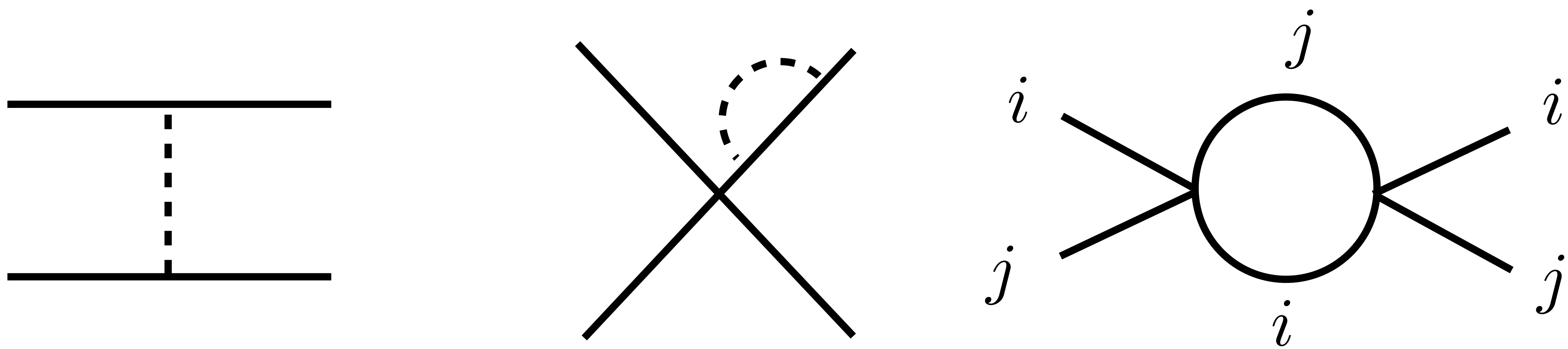


O(1/N)

Fixed point $g_*^2 = 12\pi^2\epsilon$

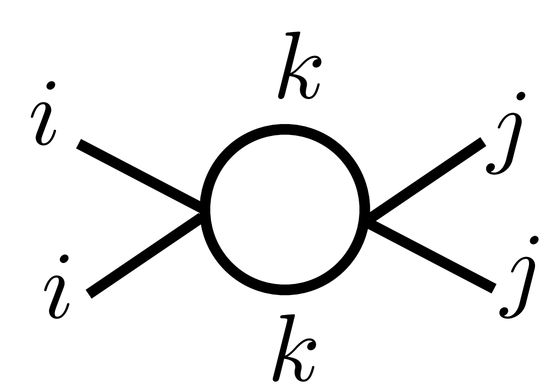
B. RG for the pairing interaction

$$L_{BCS} = \frac{v\lambda}{k_F^2 N}\,\psi_i(k)\psi_j(-k)\,\psi_i^\dagger(k')\psi_j^\dagger(-k')$$



$$\beta_\lambda = g^2 + 2\gamma\lambda + \frac{1}{2\pi^2 N}\lambda^2$$

- Approx. valid to all loops at large N and small g
- Planar diagrams subleading due to kinematics of Fermi surface



Non-Fermi liquid effects give a *3-term structure*

$$\beta_\lambda = g^2 + 2\gamma\lambda + \frac{1}{2\pi^2 N}\lambda^2$$

a) BCS case: no massless scalar $\beta_\lambda = \frac{1}{2\pi^2 N}\lambda^2$ Shankar Polchinski

For attractive interaction, $\lambda \to -\infty$ at $\mu_{\rm inst} \sim e^{-2\pi^2 N/\lambda_0}\,\Lambda$

BCS instability, superconductivity

b) "Color superconductivity": $\beta_\lambda = g^2 + \frac{1}{2\pi^2 N}\lambda^2$ Son

Parametrically enhanced instability $\mu_{\rm inst} \sim e^{-\sqrt{2\pi^2 N/g^2}}\,\Lambda$

c) Non-Fermi liquid:

$$\beta_\lambda = g^2 + 2\gamma\lambda + \frac{1}{2\pi^2 N}\lambda^2$$

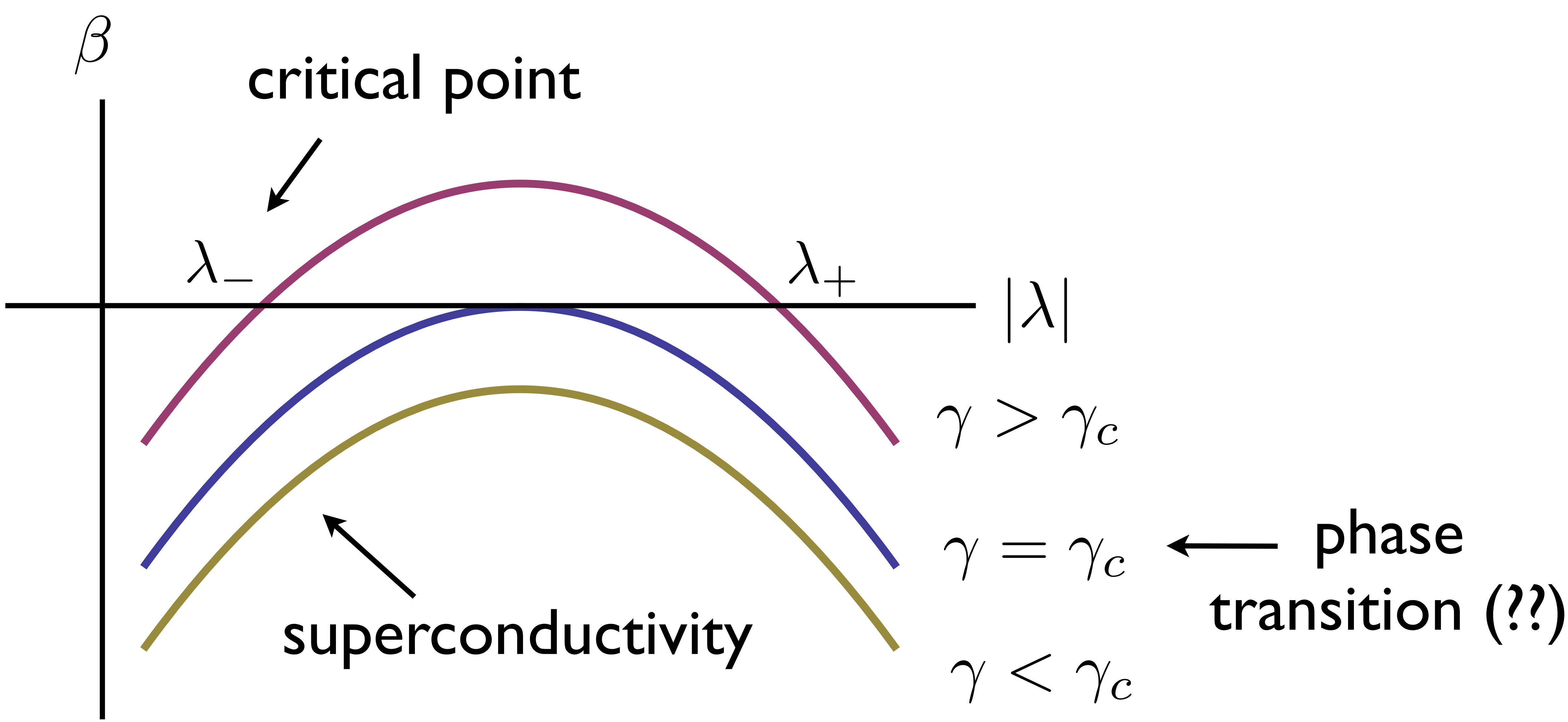
Compare scales:

$$\mu_{\rm inst} \sim e^{-\sqrt{2\pi^2 N/g^2}}\,\Lambda$$

$$\mu_{\rm NFL} \sim e^{-1/\gamma}\,\Lambda$$

$$\Rightarrow \quad \gamma_{\rm crit} = \sqrt{\frac{g^2}{2\pi^2 N}}$$

in our previous model ... $g_{\rm crit} = \dfrac{12\sqrt{2}\pi}{N}$



C. Quantum criticality

IR and UV fixed pts $$\lambda_{\pm} = -2\pi^2 N\gamma\left(1 \pm \sqrt{1-\gamma_c/\gamma}\right)$$

Focus on the stable IR critical point

- NFL effects on fermions $Z(p_0) = Z_v(p_0)^{-1} = p_0^{-2\gamma}$
- Superconducting instability is absent; 4-F pairing irrelevant
- Cooper pair op: $\mathcal{O} = \psi(k)\psi(-k) \;\Rightarrow\; \gamma_{\mathcal{O}} = 2\gamma + \frac{\lambda_*}{2\pi^2 N}$
- Approach to the fixed point: perturb $\lambda \to \lambda + \delta\lambda$

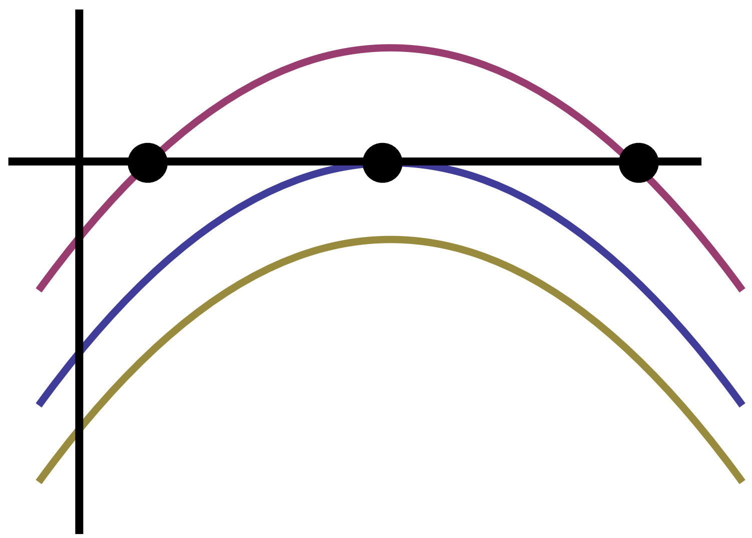
$$\beta_\lambda \approx \beta'_*(\lambda - \lambda_*) \quad \Rightarrow \quad \lambda(\mu) = \lambda_* + \delta\lambda \times (\mu/\Lambda)^{\beta'_*}$$

$$\beta'_* = 2\sqrt{\gamma^2 - \gamma_c^2}$$ ⟵ anomalous dim. of 4-Fermi operator

Phase transition

As $\gamma \to \gamma_{\rm crit}$ the critical point disappears

➡ IR and UV fixed points merge and annihilate



➡ from $\beta'_* = 2\sqrt{\gamma^2 - \gamma_{\rm crit}^2}$

it takes longer to approach the fixed point

➡ 4-Fermi operator becomes exactly marginal

Just below transition, the theory develops an instability at

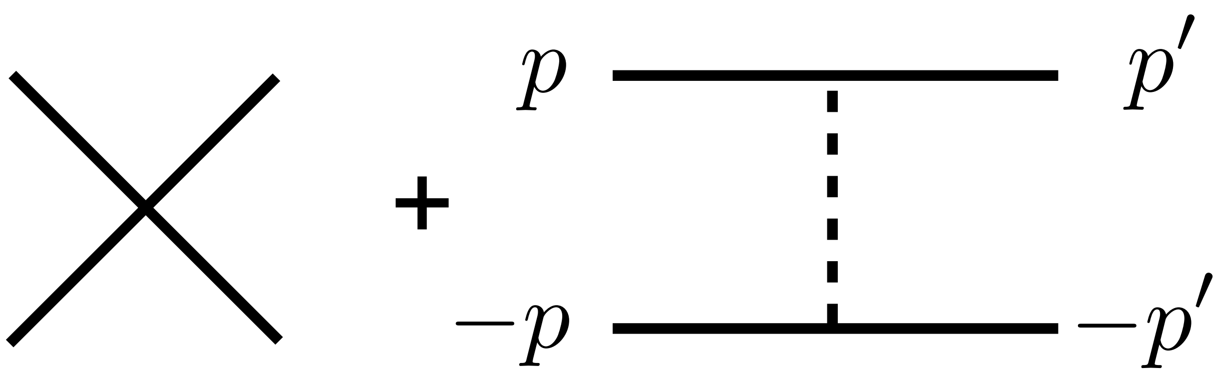
$$\mu_{\rm inst} \sim \exp\left[-\sqrt{\frac{N}{g^2}}\frac{1}{\sqrt{1-\gamma^2/\gamma_c^2}}\right]\Lambda$$

Infinite order (continuous) phase transition between QC and SC!

D. Superconductivity and NFL

Develop a framework for including NFL effects in the formation of the superconducting gap

(i) Integrate out massless scalar exactly



$$= V(p,p') = -\lambda + g^2 D(p-p')$$

(ii) Hubbard-Stratonovich on 4-F int. to introduce the SC gap

$$V\,\psi\psi\,\psi^\dagger\psi^\dagger = -V^{-1}\Delta^\dagger\Delta + \Delta^\dagger\psi\psi + \Delta\psi^\dagger\psi^\dagger$$

(iii) Integrate out fermions. Effective action for gap

$$S = \int_{p,p'} V^{-1}(p,p')\Delta^{\dagger}(p)\Delta(p') - \int_p \log\left[(Z(p)p_0)^2 + \varepsilon(p)^2 + \Delta(p)^2\right]$$

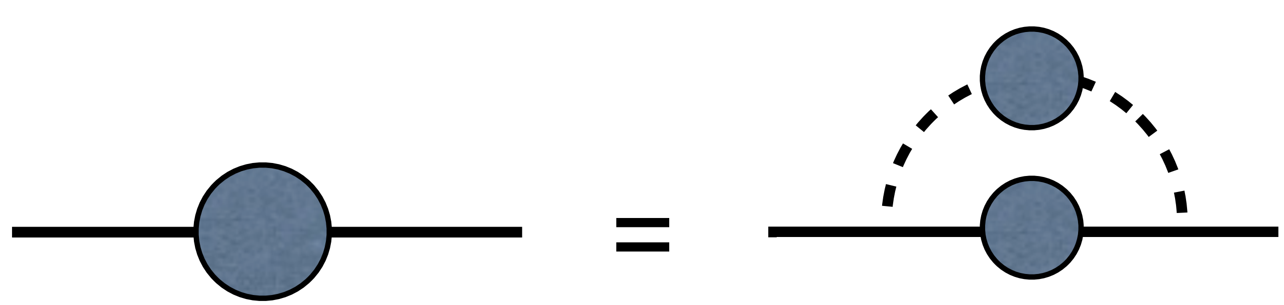
classical piece.
Nonlocal mass (positive)

fermion determinant.
Favors instability

(iv) Extremize the effective action

$$\Delta(p) = \int_q V(p,q)\frac{\Delta(q)}{Z(q_0)^2 q_0^2 + \Delta(q)^2 + \varepsilon(q)^2}$$

Classical approximation valid at large N: agrees with Schwinger-Dyson



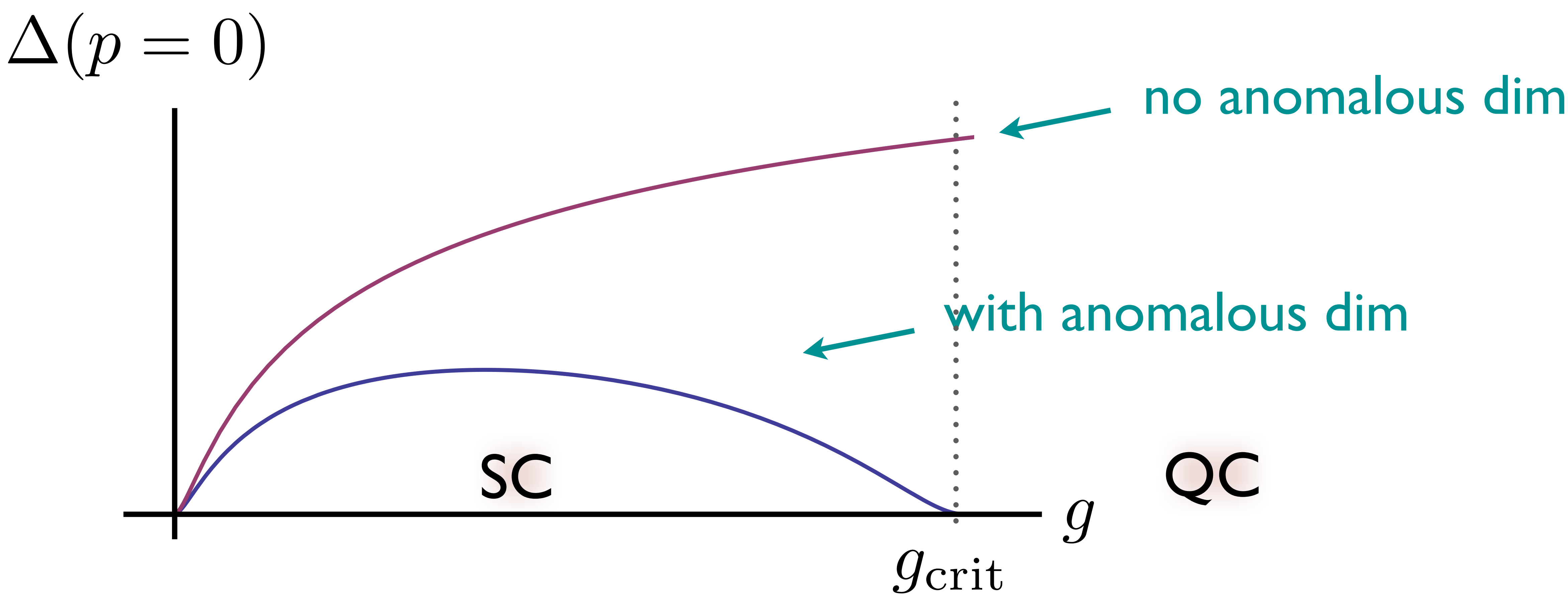
neglects vertex corrections.
OK at large N

The integral Schwinger-Dyson-Eliashberg eq. can be solved numerically.

Can also transform it (approx.) into a differential equation. [Son]

Result agrees qualitatively with RG approach.

Evolution of the physical gap:



D. Conclusions

NFL effects (anomalous dim, velocity renorm) can produce strong changes in the IR dynamics, even at weak coupling

✓ New fixed point for the 4-Fermi Cooper interaction

✓ Superconductivity with large NFL corrections

✓ Continuous phase transition connecting QCP and SC

Future directions

- Continue classification of phases of finite density QFT
- Implications for QCD at finite density
- Apply to other model theories
- Extend results to strong coupling