Interacting topological insulators in 3D

Anomalies in $(2+1)D$ interacting field theories

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Based on works with Andrew C. Potter, Adam Nahum and T. Senthil

Topological insulator 101

- Zero-temperature phases with some global symmetry G
- Bulk: gapped, excitation spectrum $\sim$ trivial insulator
- Boundary theory **anomalous**:
  cannot exists on its own, must live on the boundary
- Free fermion limit: gapless boundaries
  – massless Dirac/Weyl/Majorana fermions
  – fully classified (Ryu, Schnyder, Furusaki, Ludwig; Kitaev)

Topological insulator 101

Classic example: 3D TI with time-reversal symmetry $\mathcal{T}$ and charge conservation $U(1)$ (Fu, Kane, Mele)

- Surface: single (2-component) Dirac fermion

$$H_{eff} = \psi^\dagger \vec{p} \cdot \vec{\sigma} \psi$$

- Mass terms not allowed:
  - $m\bar{\psi}\psi$ breaks $\mathcal{T}$
  - $\Delta\psi\sigma^y\psi$ breaks $U(1)$
- "Parity anomaly" in field theory

  (Redlich, 1984; Mulligan, Burnell, 2013)

Beyond free fermions?

Qualitatively new features in interacting systems:

~~Gapless boundary~~ $\rightarrow$

- Boundaries could be gapped without symmetry breaking (fermion bilinear mass), with intrinsic topological order
  (Vishwanath, Senthil; CW, Senthil; Burnell, Chen, Fidkowski, Vishwanath)
- even including the famous free fermion TI – parity anomaly reproduced by a gapped topological order!
  (CW, Potter, Senthil; Metlitski, Kane, Fisher; Chen, Fidkowski, Vishwanath; Bonderson, Nayak, Qi, arXiv:1306.32**)

Beyond free fermions? (cont')

Qualitatively new features in interacting systems:

~~Fully classified~~ $\rightarrow$

- Reduction: some nontrivial free fermion phases become trivial
  Fidkowski, Kitaev; Tang, Wen (1D)
  Qi; Ryu, Zhang; Levin, Gu (2D)
  Kitaev; Fidkowski, Chen, Vishwanath; CW, Senthil; Metlitski, Chen, Fidkowski, Vishwanath (3D)

  You, BenTov, Xu; You, Xu (4D)

- **Enrichment** (this talk): new bulk states emerge, with no non-interacting counterparts $\sim$ new kinds of anomalies on surfaces

Focus of this talk

3D Interacting fermions with charge conservation $U(1)$ and time-reversal $\mathcal{T}$, microscopic fermions carry charge-1 and $\mathcal{T}^2 = -1$

Physically motivated:

- Materials with strong spin-orbit and *e*-*e* interaction: Pyrochlore iridates, Kondo insulators, etc.
- Time-reversal & charge-conservation are realistic and robust

Main result: $\mathbb{Z}_2$(free)$\rightarrow$ $\mathbb{Z}_2^3$(interacting)
(CW, Potter, Senthil 2014)

Outline

1. A simple example beyond free fermions
2. Classification: from $\mathbb{Z}_2$ to $\mathbb{Z}_2^3$
3. Characterization: surface magnets and superconductors
4. Topological paramagnet in frustrated spin-1 systems

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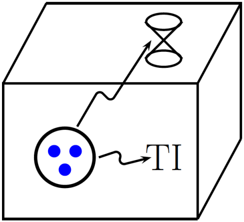
Cluster TI



- Take three-body bound states: charge-3 fermions $F \sim fff$
- Put F into a topological (Fu-Kane-Mele) band
- Surface: one flavor of charge-3 massless Dirac fermion
- Break $\mathcal{T}$ on surface:
  $\sigma_{xy} = \frac{1}{2}(e^*)^2 = \frac{9}{2}$, $\kappa_{xy} = \frac{1}{2}$, $\sigma_{xy} - \kappa_{xy} = 4$

- In strict $2D$ electron systems without topological order, $\sigma_{xy} - \kappa_{xy} = 8n$ (e.g. E_8 state with no hall conductance)
  $\rightarrow$ Cluster TI is different from free fermion TI!

  (CW, 2014)

What is it?

Claim:

Charge-3 Dirac $\cong$ Charge-1 Dirac $\otimes$ Charge-neutral $\mathbb{Z}_2$ gauge theory

Check: consider Cluster TI $\otimes$ Free TI, surface state:

$$\mathcal{L} = \bar{\psi}\sigma^{\mu}(-i\partial_{\mu} + A_{\mu})\psi + \bar{\Psi}\sigma^{\mu}(-i\partial_{\mu} + 3A_{\mu})\Psi$$

- Pairing gap (breaks $U(1)$):

$$\Delta\mathcal{L} = i\Delta\psi\sigma_y\psi + i\xi(\Delta)^3\Psi\sigma_y\Psi + h.c.$$

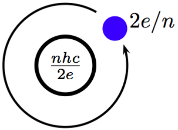
- To restore $U(1)$, need to condense vortices

What is it? (cont.)

- Vortex statistics can be calculated
  (Methods developed in CW, Potter, Senthil; Metlitski, Kane, Fisher, 2013)
  $\rightarrow$ strength-1 vortex (π-vortex) is a fermion
- Condensing strength-2 vortex (2π-vortex)
  $\rightarrow$ $\mathbb{Z}_2$ gauge theory (topological order)

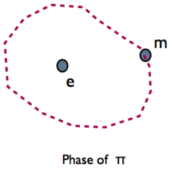
Condensing higher vortices

- "Vortex" of the $n\pi$-vortex condensate is a boson but carries charge $2/n$
- Excitations in final state:
  unconfined π vortex $+$ charge-$(2/n)$ boson b
- Only for $n = 2$: Bogoliubov fermion $e_\alpha = b^\dagger c_\alpha$ and π vortex ϵ are both charge-neutral

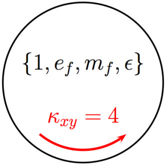


Anomalous topological order

- Excitations: Bogoliubov particle e (fermion, $\mathcal{T}^2 = -1$), un-condensed vortex ϵ (fermion), bound state $m \sim e\epsilon$ (fermion, $\mathcal{T}^2 = -1$)



- Charge-neutral $\mathbb{Z}_2$ topological order dubbed $e_f T m_f T$ –cannot be realized in strict $2D$: anomalous $\mathbb{Z}_2$ gauge theory



- Chiral if realized in strict 2D: edge chiral central charge $c = 4(\mathrm{mod} 8) \rightarrow$ must break $\mathcal{T}$

Outline

General TI surface

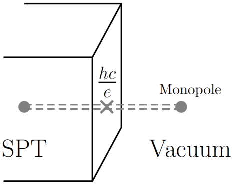
Claim: an anomalous surface is either

1. $\cong$ Charge-1 Dirac $\otimes$ Charge-neutral topological order
2. $\cong$ Charge-neutral topological order

Key step of the argument:
condensing 2π-vortices $\rightarrow$ charge quantized in integer units, and could be neutralized by binding electrons

But why can we always condense 2π-vortices?

2π-vortex from monopole tunneling



- Surface 2π-vortex can be created through monopole tunneling

  (see also Metlitski, Kane, Fisher, PRB 2013)

- First assume monopole carries no charge ($\theta = 0$)
- Time-reversal trivial (up to a gauge transform):

$$\mathcal{T} : m \leftrightarrow m^\dagger$$

- Monopole must be bosonic

  (nontrivial, for proof, see CW, Potter, Senthil, 2014, Kravec, McGreevy, Swingle, 2014)

- Monopole trivial $\rightarrow$ 2π-vortex trivial and could be condensed $\rightarrow$ surface $\cong$ Charge-neutral topological order

Charged monopole: θ-term

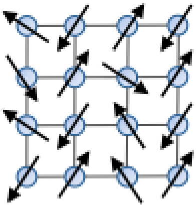
Magneto-electric response (θ-term):

$$\mathcal{L}_\theta = \frac{\theta}{4\pi^2}\vec{E}\cdot\vec{B}$$

- Witten effect: monopole charge $= \theta/2\pi$
- Time-reversal invariance $\rightarrow \theta = 0$ or π (mod 2π)
- $\theta = \pi$ for free fermion TI (Qi, Hughes, Zhang, 2008)
- If: $\theta = \pi$ for some bulk state
  combine with free fermion TI $\rightarrow \theta = 0$, monopole carries no charge
  $\rightarrow$ surface $\cong$ Charge-neutral topological order

So for $\theta = \pi$, surface
$\cong$ Charge-1 Dirac $\otimes$ Charge-neutral topological order

Anomalous charge-neutral topological orders



$$b \equiv S^- = f_\downarrow^\dagger f_\uparrow$$

$$n_b \equiv S^z = \frac{1}{2}(f_\uparrow^\dagger f_\uparrow - f_\downarrow^\dagger f_\downarrow)$$

- Anomalous charge-neutral topological orders classified by $\mathbb{Z}_2^2$
  (Chen, et al, 2011; Vishwanath, Senthil, 2012; Kapustin, 2014; Freed, 2014)
- Could also appear on the surface of spin (boson) systems: **topological paramagnets** (local objects charge neutral $\rightarrow$ must be bosonic)
- Topological paramagnets in electron systems: Mott insulators $\rightarrow$ spin (boson) systems
- Total classification $= \mathbb{Z}_2$ (Free) $\times$ $\mathbb{Z}_2^2$ (Topological paramagnets) $= \mathbb{Z}_2^3$
  (CW, Potter, Senthil, 2014)

Another topological paramagnet

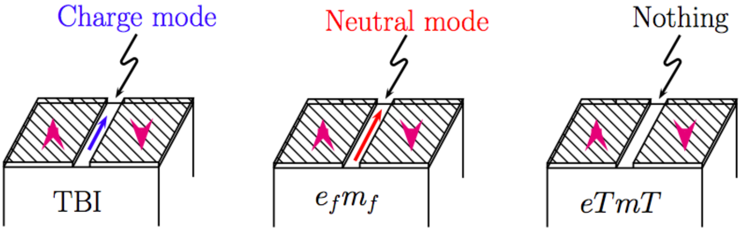
The remaining $\mathbb{Z}_2$: another topological paramagenet

- Representative surface state: $\mathbb{Z}_2$ topological order, e and m are bosons but $\mathcal{T}^2 = -1$ ($eTmT$)
- Described within group-cohomology (Chen, Gu, Liu, Wen)
- Cannot be realized in strict $2D$ – "non-edgable"

Outline

Surface magnets: hall transport

Breaking $\mathcal{T}$ (depositing a magnet): gapped without topological order

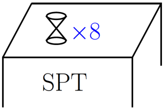


(Vishwanath, Senthil, 2012)

Surface superconductors: Majorana cones

Breaking $U(1)$ (depositing a superconductor): topological orders confined

- TBI: surface superconductors gapped
- $e_f T m_f T \cong$TBI$\oplus$Cluston TI: also gapped
- $eTmT$: surface superconductor gapless! Eight Majorana cones protected by $\mathcal{T}$
  ($\sim$ 8 copies of He-3B, but bulk can be insulating only in interacting system)



can be seen through ARPES!

Understanding the cones

Argue in reverse: start from eight Majorana cones (four Dirac)

$$H = \sum_{i=1}^{4} \psi_i^\dagger (p_x \sigma^x + p_y \sigma^z) \psi_i$$

with $\mathcal{T}$: $\psi_i \to i\sigma_y \psi_i^\dagger$

- Quadratic gaps forbidden by $\mathcal{T}$ – non-perturbative gap?
- Introduce auxiliary $U(1)$: $\psi_i \to e^{i\theta}\psi_i$
- $H_\Delta = i\Delta(\vec{x})\psi_i \sigma_y \psi_i + h.c.$
  $\to$ breaks $\mathcal{T}$ and $U(1)$, but preserves $\tilde{\mathcal{T}} = \mathcal{T} U_{\pi/2}$

Understanding the cones (cont.)

- Disorder $\Delta(\vec{x})$ ($\langle\Delta(\vec{x})\rangle = 0$) to recover symmetries
- Need to proliferate vortices, but fundamental vortex nontrivial: $\tilde{\mathcal{T}}^2 = -1$
- Condense strength-2 vortex $\rightarrow$ $\mathbb{Z}_2$-ordered $eTmT$ state!

  (related results: Fidkowski, Chen, Vishwanath, 2013; Metlitski, Fidkowski, Chen, Vishwanath, 2014)

Outline

A useful equivalence

Four Dirac cones ($\mathcal{T}$: $\psi_i \to i\sigma_y \psi_i^\dagger$) $\cong$ $eTmT$ topological order

- Also true with $SU(2)$ invariance: $\psi_{\alpha,a} \to U_{\alpha\beta}\psi_{\beta,a}$ ($a = 1, 2$)
  in particular, e and m are $SU(2)$-singlets
- This is the surface of a singlet topological superconductor at $\nu = 2$
  (Ryu, Schnyder, Ludwig)

Useful for constructing trial wavefunctions for
topological paramagnet in spin models
(CW, Nahum, Senthil, 2015)

Construction: field theory

- Start from singlet topological superconductor at $\nu = 2$
- Couple the fermions (bulk and boundary) to an $SU(2)$ gauge field
- Bulk: $SU(2)$ gauge field has a θ-term with $\theta = \nu\pi = 2\pi \rightarrow$ confines without breaking $\mathcal{T} \rightarrow$ a symmetric bulk with only bosonic d.o.f.
- Surface: e and m are $SU(2)$-singlets and decouple with the gauge field $\rightarrow eTmT$ topological order survives confinement

$\rightarrow$ a topological paramagnet in bosonic (spin) systems

Construction: slave particles

- Write spin operators (spin-1) as

$$\vec{S} = \frac{1}{2} \sum_{a=1,2} f^{\dagger}_{a\alpha} \vec{\sigma}_{\alpha\beta} f_{a\beta}$$

- Sp(4) gauge redundancy (Xu et. al, 2011)
- Consider mean field state such that
  1. Gauge group broken down to $SU(2)$
  2. $f_{a\alpha}$ form $SU(2)$-singlet superconductors at $\nu = 2$
- Projecting into physical Hilbert space $\rightarrow$ introducing gauge fluctuation $\rightarrow$ bulk confined

$\rightarrow$ a trial projective wavefunction of topological paramagnet in frustrated spin-1 systems

Summary

- $\mathbb{Z}_2$ (Free) $\times$ $\mathbb{Z}_2^2$ (Topological paramagnets) $=$ $\mathbb{Z}_2^3$ (Interacting)
- Topological paramagnet (possibly) in frustrated spin-1 systems

Thank you!