Interacting Topological Insulator and the Grand Unified Theory

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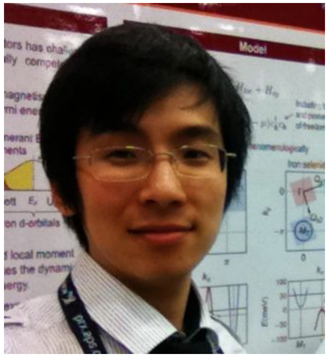
Collaborators:
Postdoc: Yi-Zhuang You

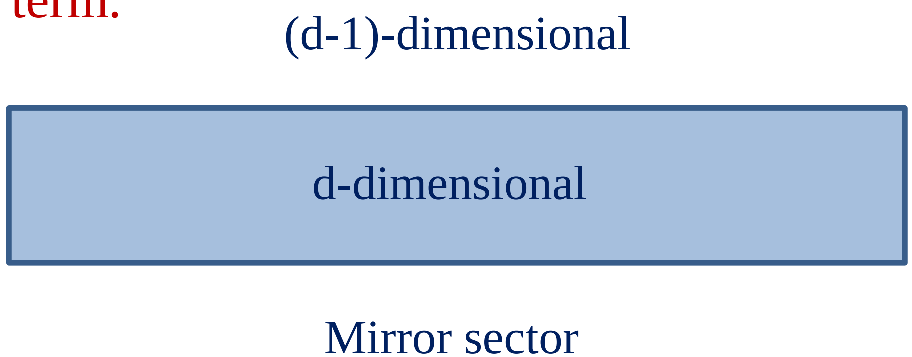


Very helpful discussions with
Joe Polchinski, Mark Srednicki, Robert Sugar, Xiao-Gang Wen, Alexei Kitaev, Tony Zee…….

Reference: You, Xu, arXiv:1412.4784

Topological Insulator 101:

d-dimensional bulk: massive Dirac/Majorana fermion;
(d-1)-dimensional boundary: gapless Dirac/Weyl/Majorana fermions, gapless spectrum protected by symmetry, i.e. Symmetry forbids fermion mass term.



(d-1)-dimensional boundary cannot exist as a (d-1)-dimensional system without the bulk. i.e. Once symmetries are gauged, will have gauge anomaly. Full classification of noninteracting topological insulator: (Ryu, et.al., Kitaev, 2009)

Topological Insulator 101:

The boundary of TI without any symmetry must have gravitational anomaly.

Example: topological superconductor with no symmetry at all

Classification (Ryu, et.al., Kitaev, 2009)

d	1	2	3	4	5	6	7	8	9	10	11
	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

Gravitational Anomaly of single Majorana fermion (Alvarez-Gaume, Witten, 1983)

$d-1$	1	2	3	4	5	6	7	8	9	10
	P				P		G	G	P	

Topological Insulator 101:

The boundary of TI with unitary symmetry G will have gauge anomaly once G is "gauged".

Example: topological insulator with U(1) symmetry

Classification (Ryu, et.al., Kitaev, 2009)

d	1	2	3	4	5	6	7	8	9	10	11
	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	

U(1) gauge anomaly at the boundary:

$d-1$	1	2	3	4	5	6	7	8	9	10
	P		P		P		P		P	

Topological Insulator 101:

The boundary of TI with unitary symmetry G will have gauge anomaly once G is "gauged".

Example: topological superconductor with SU(2) symmetry

Classification (Ryu, et.al., Kitaev, 2009)

d	1	2	3	4	5	6	7	8	9	10	11
	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0

SU(2) gauge anomaly at the boundary:

$d-1$	1	2	3	4	5	6	7	8	9	10
	P		G	G	P				P	

Interacting Topological Insulator and the Grand Unified Theory

Motivation:

1. Finding an application for interacting symmetry protected topological states, especially a non-industry application;

2. Many high energy physicists are studying CMT using high energy techniques, we need to return the favor.

Current understanding of interacting TI:
Interaction may not lead to any new TI, but it can definitely "reduce" the classification of TI, i.e. interaction can drive some noninteracting TI trivial, in other words, interaction can gap out the boundary of some noninteracting TI, without breaking any symmetry,
Or equivalently, interaction can gap out the boundary without generating a fermion mass term.

Weyl fermions:

$$H_L = \psi_L^\dagger (i\vec{\sigma} \cdot \vec{\partial})\psi_L$$

$$H_R = \psi_R^\dagger (-i\vec{\sigma} \cdot \vec{\partial})\psi_R$$

$$\tilde{\psi}_L = \sigma^y \psi_R^\dagger$$

$$H_R = \tilde{\psi}_L^\dagger (i\vec{\sigma} \cdot \vec{\partial})\tilde{\psi}_L$$

Weyl fermions can be gapped out by Cooper pairing (Majorana mass):

$$H_m = m\psi_L \sigma^y \psi_L + h.c.$$

Regularizing Grand Unified Theories on a lattice

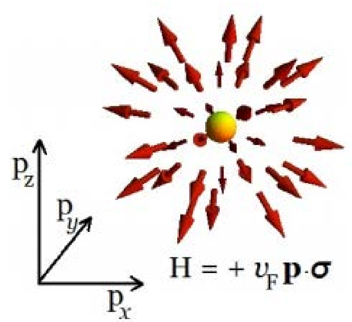
Very high energy In Standard Model (higher than EW unification energy), every generation has (effectively) 16 massless Left Weyl fermions coupled with gauge field: (spinor rep of SO(10))

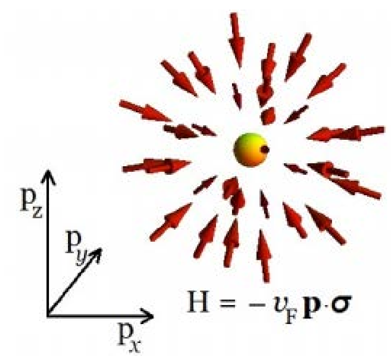
$$(u_\alpha, d_\alpha)_L,\ (u_\alpha^\dagger, d_\alpha^\dagger)_R,\ (e, \nu_e)_L,\ (e^\dagger, \nu_e^\dagger)_R$$

$$2\times 3 + 2\times 3 + 2 + 2 = 16$$

This theory is difficult to regularize on a 3d lattice. Because on a 3d lattice, if we want to realize left fermions, we also get right fermions coupled to the same gauge theory

For example: Weyl semimetal has both left, and right Weyl fermions in the 3d BZ:





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This theory is difficult to regularize on a 3d lattice. Because on a 3d lattice, if we want to realize left fermions, we also get right fermions coupled to the same gauge theory

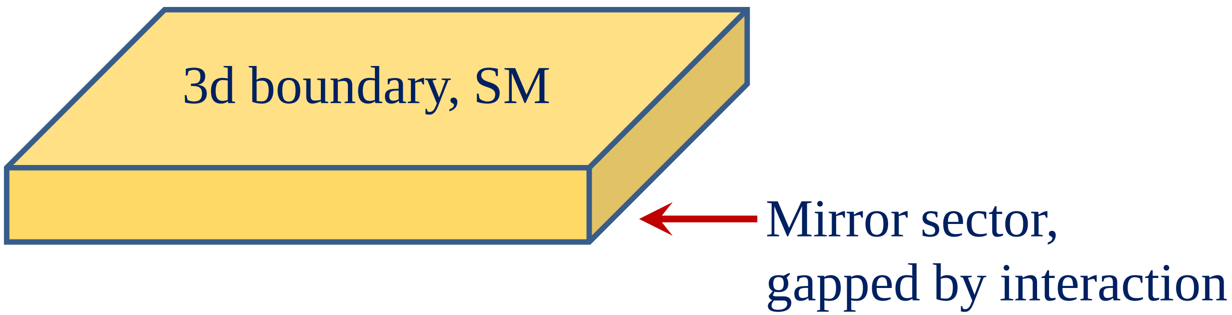
Popular alternative: Realize Weyl fermions on the 3d boundary of a 4d topological insulator/superconductor

However, this approach requires a subtle adjustment of the fourth dimension. If the fourth dimension is too large, there will be gapless photons in the bulk; if the fourth dimension is too small, the mirror sector on the other boundary will interfere.

Key question: Can we gap out the mirror sector (Weyl fermions on the other boundary) without affecting the SM at all?

This cannot be done in the standard way (spontaneous symmetry breaking, condense a boson that couples to the mirror fermion mass)

$$\phi \psi_R^t i\sigma^y \psi_R \longrightarrow \phi \psi_L^t i\sigma^y \psi_L$$



A different question: Can we gap out the mirror sector with short range interaction, while

$$\langle \psi_R^t i\sigma^y \psi_R \rangle = 0$$

If this is possible, then only the SM survives at low energy.

All we need to do, is argue that the 4d bulk topological insulator/superconductor is nontrivial without interaction, but trivialized by interaction.
Similar logic by Wen, but very different way of dealing with interacting TI from us.

ABSENCE OF NEUTRINOS ON A LATTICE
(I). Proof by homotopy theory

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The assumptions made for the charges Q (lepton number, say) of the theory are the following:

(i) Exact conservation of Q, even at scales where the lattice cutoff is relevant. Charge conservation means that the energy and momentum eigenstates are also charge eigenstates.

This means, in order to realize Weyl fermions without “mirror sector”, we must break the (anomalous) U(1) symmetry explicitly. The U(1) symmetry becomes an emergent symmetry at IR.

0d boundary of 1d TSC

Consider N copies of 0d Majorana fermions with time-reversal symmetry (boundary of N copies of Kitaev's 1d TSC):

$$T : \gamma_a \to \gamma_a ,$$

$i\gamma_a\gamma_b$ Breaks time-reversal

For N = 2, the only possible Hamiltonian is

$$H = Ki\gamma_1\gamma_2$$

But it breaks time-reversal symmetry, thus with time-reversal symmetry, H = 0, the state is 2-fold degenerate.

For N = 4, the only T invariant Hamiltonian is

$$H = K\gamma_1\gamma_2\gamma_3\gamma_4 = -\frac{K}{4}(2i\gamma_1\gamma_2)(2i\gamma_3\gamma_4) \sim -\frac{K}{4}\sigma_1^z\sigma_2^z$$

Finally, when N = 8,

$\gamma_{1,2,3,4}$ $\gamma_{5,6,7,8}$

doublet doublet

$$H = \vec{S}_1 \cdot \vec{S}_2$$

$$\downarrow$$

GS fully gapped,
nondegenerate

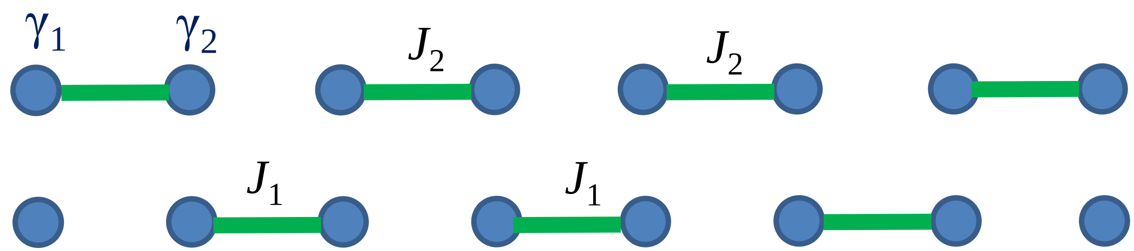
Thus, when N = 8, the Majorana fermions can be gapped out by interaction without degeneracy, and

$$\langle i\gamma_a\gamma_b\rangle = 0$$

These 0d fermions are realized at the boundary of 1d TSC:

Trivial

TSC



$$H=\sum_j iJ_1\gamma_{2j}\gamma_{2j+1}+iJ_2\gamma_{2j+1}\gamma_{2j+2}$$

$$J_1>J_2,\quad \text{topological},\qquad J_2>J_1,\quad \text{trivial},$$

In the bulk: $T:\ \gamma_{2j}\to-\gamma_{2j},\qquad \gamma_{2j+1}\to\gamma_{2j+1}$

With *N* flavors, at the boundary $T:\gamma_a\to\gamma_a,$

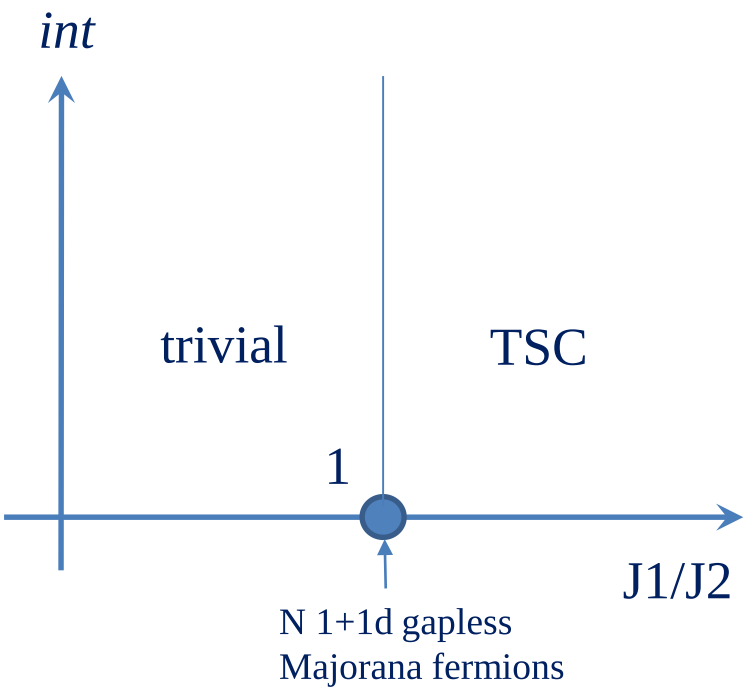
This implies that, with interaction, 8 copies of such 1d TSC is trivial, i.e. interaction reduces the classification from **Z** to $\mathbf{Z_8}$.
Fidkowski, Kitaev, 2009

0d boundary of 1d TSC

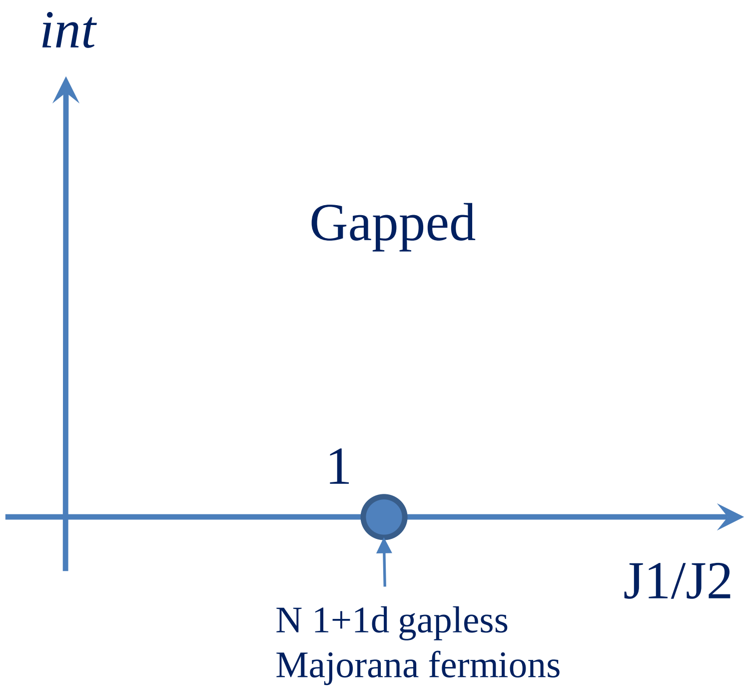
$$H = \sum_j iJ_1 \gamma_{2j}\gamma_{2j+1} + iJ_2 \gamma_{2j+1}\gamma_{2j+2}$$

$$J_1 > J_2, \quad \text{topological}, \qquad J_2 > J_1, \quad \text{trivial},$$

For N flavors with N = 8n+k



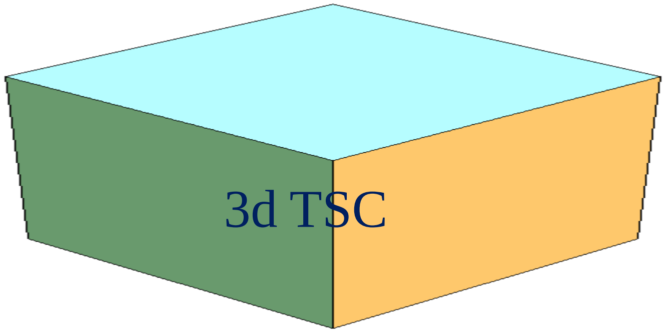
For N flavors with N = 8n,
Fidkowski, Kitaev, 2009



2d boundary of 3d TSC

$$H_0 = \int d^3x \sum_{a=1}^{N} \chi_a^\intercal (\mathrm{i}\Gamma^1\partial_x + \mathrm{i}\Gamma^2\partial_y + \mathrm{i}\Gamma^3\partial_z + m\Gamma^4)\chi_a$$

$$\Gamma^1 = \sigma^{30},\ \Gamma^2 = \sigma^{10},\ \Gamma^3 = \sigma^{22},\ \Gamma^4 = \sigma^{21},\ \Gamma^5 = \sigma^{23}$$



$$H_{edge} = \int d^2x \ \sum_a \chi_a^t (i\tau^x\partial_x + i\tau^z\partial_y)\chi_a \qquad T: \chi_a \to i\tau^y\chi_a$$

Short range interactions reduce the classification of the 3d TSC from **Z** to $\mathbf{Z}_{16}$, namely its edge (16 copies of 2d Majorana fermions) can be gapped out by interaction, with $\langle\bar{\chi}\chi\rangle = 0$
(Vishwanath, et.al. 2014, Kitaev, and other groups)

2d boundary of 3d TSC

Consider a modified boundary Hamiltonian (Wang, Senthil 2014):

$$H = \int d^2x \sum_{a=1,2} \chi_a^t (i\tau^x \partial_x + i\tau^z \partial_y)\chi_a + \phi_x \chi^t \tau^y \sigma^x \chi + \phi_y \chi^t \tau^y \sigma^z \chi$$

Consider an enlarged O(2) symmetry.
When ϕ condenses/orders, it breaks T, breaks O(2), but keeps

$$T' = T \otimes (\pi\text{-rotation})$$

All the symmetries can be restored by condensing the vortices of the ϕ order parameter. A fully gapped, nondegenerate, symmetric state is only possible if the vortex is gapped, nondegenerate.
A vortex core has one Majorana mode, and

$$T' : \gamma_a \to \gamma_a$$

With N = 16, interaction can gap out the 2d boundary with no deg.

2d boundary of 3d TSC

Dual theory for vortices and U(1) Goldstone mode:

$$\mathcal{L}_{dual} = \sum_{a=1}^{n} |(\partial_\mu - i a_\mu) v_a| + r|v_a|^2 + \cdots$$

When vortices are gapped, gauge field gapless, dual to the the Goldstone mode of O(2) order parameter.

If vortex core is degenerate, dual theory is the CP^{n-1} theory, condensate of vortices must be degenerate.

With 16 copies of this 3d TSC, the vortex core at the boundary is gapped and nondegenerate. Thus we can condense the vortex, restore the symmetry, keep the spectrum gapped. The fermions acquire a local four-fermion interaction after integrating out ϕ.

3d boundary of 4d TSC (Toy model of SM)

The 3d boundary of a 4d TSC with U(1) x T x Z2 symmetry:

$$H = \int d^3x \ \sum_{a=1}^{2} \psi_a^\dagger (i\vec{\sigma} \cdot \vec{\partial}) \psi_a$$

These symmetries guarantee that no quadratic mass terms are allowed at the 3d boundary. So without interaction the classification of this 4d TSC is **Z**.

We want to argue that, with interaction, the classification is reduced to $\mathbf{Z_8}$, namely the interaction can gap out **16** flavors of 3d left chiral fermions without generating any quadratic fermion mass.

3d boundary of 4d TSC (Toy model of SM)

The 3d boundary of a 4d TSC with U(1) x T x Z2 symmetry:

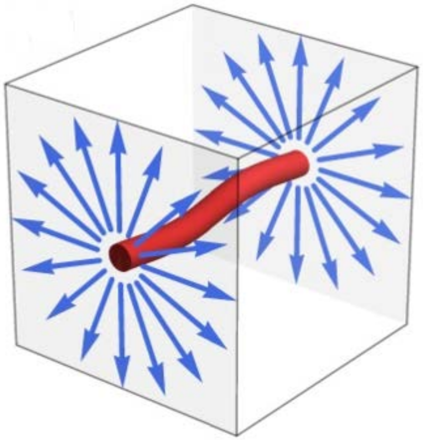
$$H = \int d^3x \sum_{a=1}^{2} \psi_a^\dagger (i\vec{\sigma} \cdot \vec{\partial}) \psi_a$$

Now consider U(1) order parameter:

$$\vec{\phi} = (\mathrm{Re}[\psi^t \sigma^y \tau^x \psi], \quad \mathrm{Re}[\psi^t \sigma^y \tau^z \psi])$$

U(1) symmetry can be restored by proliferating vortex loop.

$$H_{vortex} = \int dx \; \chi_L i\partial_x \chi_L - \chi_R i\partial_x \chi_R \qquad Z_2 : \begin{matrix} \chi_L \to \chi_L , \\ \chi_R \to -\chi_R \end{matrix}$$



For N=1 copy, the vortex line is a gapless 1+1d Majorana fermion with T and Z2 symmetry when and only when N=8 (16 chiral fermions at the 3d boundary), interaction can gap out vortex loop without degeneracy.

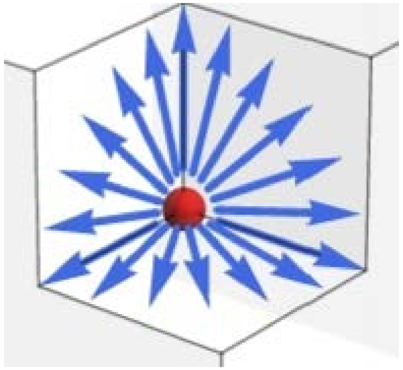
3d boundary of 4d TSC (Toy model of SM)

The 3d boundary of a 4d TSC with U(1) x T x Z2 symmetry:

$$H = \int d^3x \sum_{a=1}^{2} \psi_a^\dagger (i\vec{\sigma} \cdot \vec{\partial}) \psi_a$$

Now consider three component order parameter:

$$\vec{\phi} = \mathrm{Re}\left[\psi^t (\sigma^y \otimes i\tau^y \vec{\tau}) \psi\right]$$



All the symmetries can be restored by condensing the hedgehog monopole of the order parameter. For N=1 copy, the monopole is a 0d Majorana fermion with T symmetry

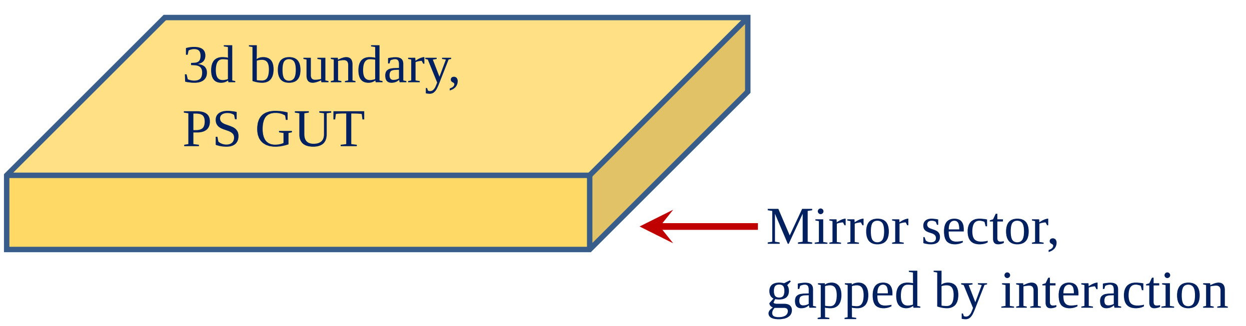
$$T : \gamma_a \to \gamma_a,$$

Then when N=8 (16 chiral fermions at the 3d boundary), interaction can gap out monopole.

Pati-Salam Grand Unified Theory

16 left handed chiral fermions decompose into representations of $SU(4) \times SU(2)_1 \times SU(2)_2$ gauge groups as (4, 2, 1) and (4*, 1, 2)
For more details, please wiki.

Our goal is to argue that, a 4+1d topological insulator (superconductor) with $SU(4) \times SU(2)_1 \times SU(2)_2$ symmetry, whose boundary has 16 chiral fermions, without interaction has Z classification, but under interaction becomes trivial.



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Our goal is to argue that, a 4+1d topological insulator (superconductor) with $SU(4) \times SU(2)_1 \times SU(2)_2$ symmetry, whose boundary has 16 chiral fermions, without interaction has Z classification, but under interaction becomes trivial.

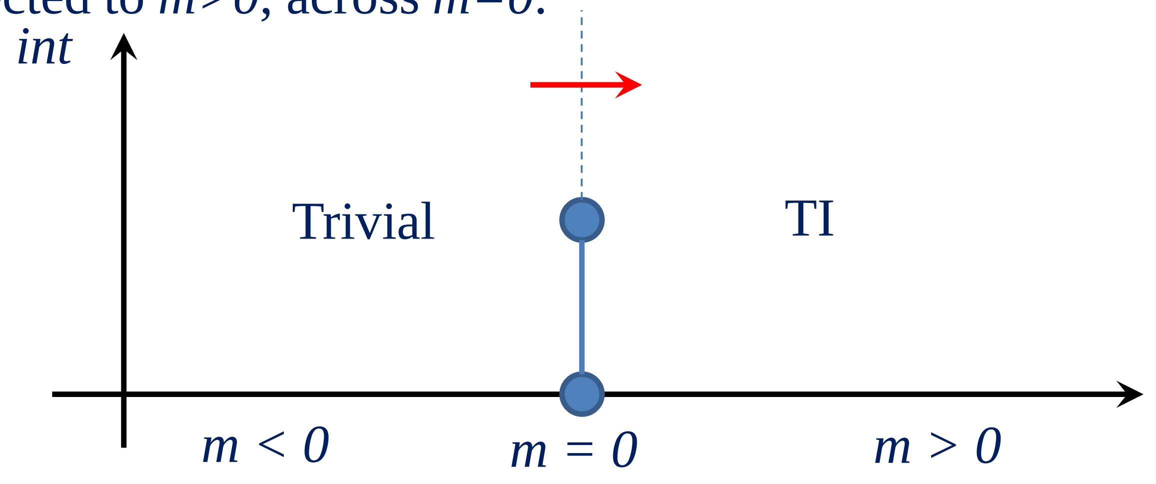
To make this statement, there are two strategies:
1, argue the boundary can be gapped out without generating any fermion bilinear mass;
2, argue the bulk quantum critical point between trivial and TI phases can be gapped out by interaction; i.e. there is only one trivial phase in the bulk.
We will take the second strategy.

Pati-Salam Grand Unified Theory

Consider a 4+1d TI or TSC with SU(4) x $SU(2)_1$ x $SU(2)_2$ symmetry. Without interaction, it is 16 flavors of 4+1d integer quantum Hall state.

$$H = \int d^4x \sum_{a=1}^{16} \psi_a^\dagger \left(\sum_{j=1}^{4} i\Gamma^j \partial_j + m\Gamma^5 \right) \psi_a$$

Goal: argue that the *m=0* line can be gapped by interaction with SU(4) x $SU(2)_1$ x $SU(2)_2$ symmetry, and state *m<0* can be smoothly connected to *m>0*, across *m=0*.



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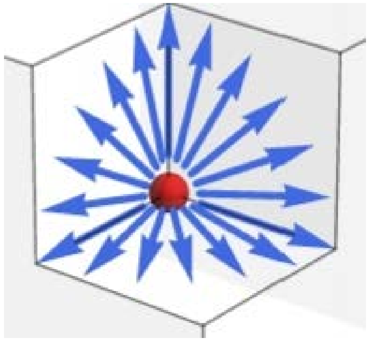
$$H = \int d^4x \sum_{a=1}^{16} \psi_a^\dagger (\sum_{j=1}^{4} i\Gamma^j \partial_j + m\Gamma^5)\psi_a$$

Strategy (sketch):
spontaneously break $SU(2)_1$ x $SU(2)_2$ symmetry by cooper pair. The cooper pair will be an SO(4) vector. Recall SO(4) ~ $SU(2)_1$ x $SU(2)_2$

$$(4,2,1) \otimes (\bar{4},1,2) = (1,2,2) \oplus \cdots$$

The SO(4) symmetry can be restored by condensing the monopoles of the SO(4) vector. We need to argue, while tuning *m*, the spectrum of monopole never closes gap when and only when there is interaction.

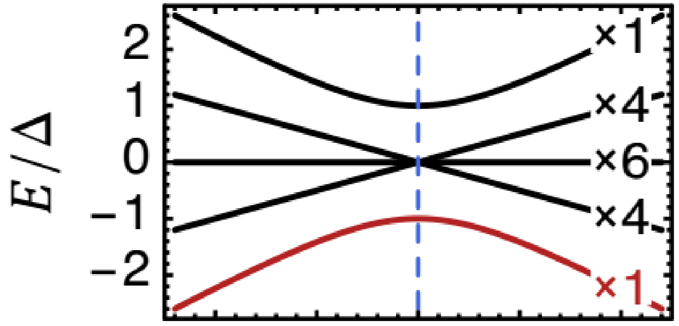


The spectrum of monopole: four localized fermion modes, $f_1 \ldots f_4$.

Effective Hamiltonian in the monopole core:

$$H_{eff} \sim m(\sum_{a=1}^{4} f_a^\dagger f_a - 2) - U(f_1 f_2 f_3 f_4 + H.c.)$$

The monopole spectrum changes smoothly without closing gap by tuning m, thus the system never closes gap after condensing monopoles, and restoring all the symmetries.



$m < 0$ $m = 0$ $m > 0$

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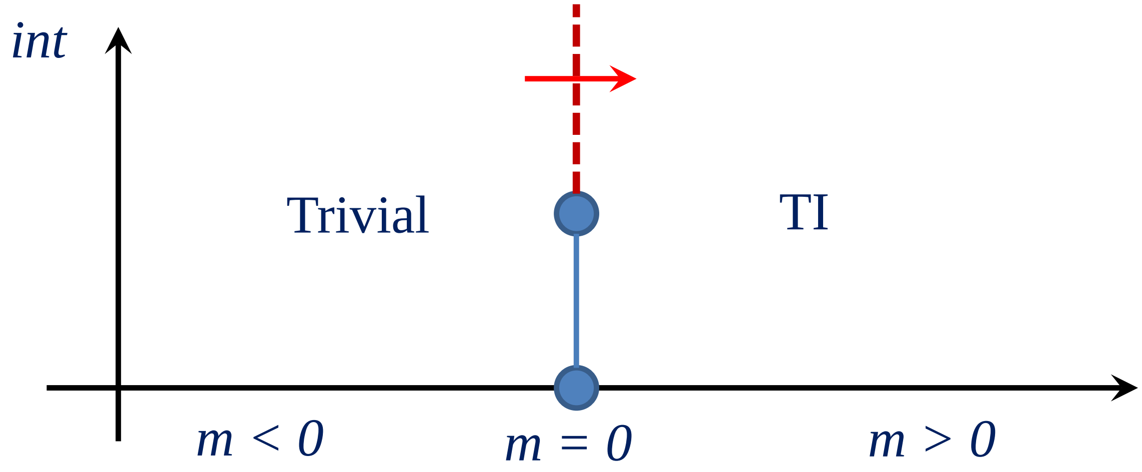
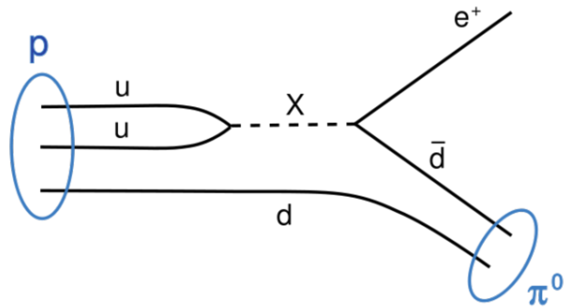


Pati-Salam Grand Unified Theory

Prediction: if this mechanism is going to work, the four fermion interaction necessarily breaks the global (anomalous) U(1) symmetry down to Z_4. i.e. the universe is a charge-4e SC.

$$H_{eff} \sim m(\sum_{a=1}^{4} f_a^\dagger f_a - 2) - U(f_1 f_2 f_3 f_4 + H.c.)$$

$$\frac{qqql}{\Lambda^2}$$



This four fermion interaction will lead to proton decay at low energy. But, weaker than proton decay of SU(5) GUT.

The detailed 4d lattice Hamiltonian can be found at
You, Xu, arXiv:1412.4784