A quantum dimer model for the pseudogap metal

Andrea Allais

Harvard University Physics Department

Aspen Center for Physics - February 20, 2015

Slides online: andrea-allais.rhcloud.com
Goal:

- describe a simple model
- hosting a fractionalized Fermi liquid phase
- possibly realized in the pseudogap phase of cuprates
Collaborators

Matthias Punk
(Innsbruck)

Debanjan Chowdhury
(Harvard)

Subir Sachdev
(Harvard)

Slides online: andrea-allais.rhcloud.com
Cuprates

- Layered, quasi-2d materials
- Layers: decorated copper-oxygen square lattice
- Most interesting physics: less than 1 electron per site.
- Well described by Hubbard model
Hubbard model

\[ H = \sum_r \left[ \sum_a -t_a c_r^\dagger a c_r + \frac{1}{2} U n_r (n_r - 1) \right] \]
Hubbard model

\[ H = \sum_r \left[ \sum_a -t_a c_r^\dagger a c_r + \frac{1}{2} U n_r (n_r - 1) \right] \]
Hubbard model

\[ H = \sum_r \left[ \sum_a -t_a c^\dagger_r a c_r + \frac{1}{2} U n_r (n_r - 1) \right] \]

Slides online: andrea-allais.rhcloud.com
Hubbard model

\[ H = \sum_r \left[ \sum_a -t_a c_r^{\dagger} a c_r + \frac{1}{2} U n_r (n_r - 1) \right] \]
$t$-$J$ model

$$H = \sum_{r,a} \left[ -t_a c_{r+a}^{\dagger} c_r + \frac{1}{2} J_a S_r \cdot S_{r+a} \right]$$
Cuprates phase diagram

Antiferromagnet (AF):

Slides online: andrea-allais.rhcloud.com
Cuprates phase diagram

Pseudogap (PG):

- Portions of the Fermi surface are gapped
- No apparent order
Cuprates phase diagram

Pseudogap (PG):

- Portions of the Fermi surface are gapped
- No apparent order

Cuprates phase diagram

**Pseudogap (PG):**

Thermal fluctuations of low temperature orders

OR

New metallic state with topological order

K. Shen *et al.*

Spin liquid

- Paramagnetic state with no long range order
- Emergent gauge field
- Liquid state: correlations decay fast
- Pictorially: superposition of many spin singlet pair configurations
Spin liquid

\[ \frac{1}{\sqrt{2}} \begin{bmatrix} \uparrow & \uparrow \\ \downarrow & \downarrow \end{bmatrix} \]

- Paramagnetic state with no long range order
- Emergent gauge field
- Liquid state: correlations decay fast
- Pictorially: superposition of many spin singlet pair configurations

Slides online: andrea-allais.rhcloud.com
Spin liquid

- Paramagnetic state with no long range order
- Emergent gauge field
- Liquid state: correlations decay fast
- Pictorially: superposition of many spin singlet pair configurations

\[ = \frac{1}{\sqrt{2}} \left[ \begin{array}{c} \hline \hline \end{array} \right] \]
Spinons

- Create triplet excitation
- Pay energy cost

\[
\begin{bmatrix}
\begin{array}{c}
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\end{array}
\end{bmatrix}
= \frac{1}{\sqrt{2}} \left[
\begin{bmatrix}
\begin{array}{c}
\uparrow \\
\downarrow \\
\uparrow \\
\downarrow \\
\end{array}
\end{bmatrix}
- \begin{bmatrix}
\begin{array}{c}
\uparrow \\
\downarrow \\
\uparrow \\
\downarrow \\
\end{array}
\end{bmatrix}
\right]
\]
Spinons

Create triplet excitation
Pay energy cost
Spins can wander away at little to no extra cost

\[ \equiv \frac{1}{\sqrt{2}} \left[ \begin{array}{ccc} & & \uparrow \\ \uparrow & & \uparrow \\ & & \downarrow \end{array} \right. - \left. \begin{array}{ccc} & & \downarrow \\ \downarrow & & \downarrow \\ & & \uparrow \end{array} \right] \]
Spinons

\[ \frac{1}{\sqrt{2}} \left[ \begin{array}{c} \uparrow \\ \downarrow \\ \end{array} \right] - \left[ \begin{array}{c} \downarrow \\ \uparrow \\ \end{array} \right] \]

- Create triplet excitation
- Pay energy cost
- Spins can wander away at little to no extra cost

Slides online: andrea-allais.rhcloud.com
Spinons

- Create triplet excitation
- Pay energy cost
- Spins can wander away at little to no extra cost

\[ \frac{1}{\sqrt{2}} \left( \begin{array}{c} \uparrow \downarrow \hline \downarrow \uparrow \end{array} - \begin{array}{c} \downarrow \uparrow \hline \uparrow \downarrow \end{array} \right) \]
Spinons

\[ \frac{1}{\sqrt{2}} \left[ \begin{array}{l}
\vdots \\
\vdots \\
\vdots \\
\vdots
\end{array} \right] \]

- Create triplet excitation
- Pay energy cost
- Spins can wander away at little to no extra cost
- Neutral, spin 1/2 excitation, gauge-charged: spinon

Slides online: andrea-allais.rhcloud.com
Electron fractionalization and holons

\[ \frac{1}{\sqrt{2}} \left[ \begin{array}{c} \uparrow \downarrow \\ \downarrow \uparrow \end{array} \right] \]

- Take out an electron
Electron fractionalization and holons

- Take out an electron
- Spin and charge degrees move independently

\[ \frac{1}{\sqrt{2}} \left[ \begin{array}{c} \uparrow \downarrow \\ \downarrow \uparrow \end{array} \right] \]
Electron fractionalization and holons

- Take out an electron
- Spin and charge degrees move independently

\[ \frac{1}{\sqrt{2}} \left[ \begin{array}{c} \uparrow \rightarrow \downarrow \rightarrow \rightarrow \uparrow \\ \downarrow \rightarrow \uparrow \rightarrow \rightarrow \downarrow \end{array} \right] \]
Electron fractionalization and holons

- Take out an electron
- Spin and charge degrees move independently

\[ \frac{1}{\sqrt{2}} \left[ \begin{array}{c} \vdots \\ \bullet \end{array} - \begin{array}{c} \vdots \\ \bullet \end{array} \right] \]
Electron fractionalization and holons

- Take out an electron
- Spin and charge degrees move independently

\[ \text{\bullet} = \frac{1}{\sqrt{2}} \left( \text{\uparrow} \text{\downarrow} - \text{\downarrow} \text{\uparrow} \right) \]
Electron fractionalization and holons

- Take out an electron
- Spin and charge degrees move independently
- Hole fractionalized into spinon and gauge-charged, spinless charge $+e$ excitation: holon

\[
\frac{1}{\sqrt{2}} \left[ \begin{array}{c} \uparrow \downarrow \\ \downarrow \uparrow \end{array} \right]
\]
Electron fractionalization and holons

- Take out an electron
- Spin and charge degrees move independently
- Hole fractionalized into spinon and gauge-charged, spinless charge $+e$ excitation: holon
- Problem: no coherent quasiparticle with electron quantum numbers
Electron fractionalization and holons

- Take out an electron
- Spin and charge degrees move independently
- Hole fractionalized into spinon and gauge-charged, spinless charge $+e$ excitation: holon
- Problem: no coherent quasiparticle with electron quantum numbers

$$\frac{1}{\sqrt{2}} \left[ \begin{array}{c} \text{spinon} \\ \text{holon} \end{array} \right]$$

Slides online: andrea-allais.rhcloud.com
Electron fractionalization and holons

- Take out an electron
- Spin and charge degrees move independently
- Hole fractionalized into spinon and gauge-charged, spinless charge $+e$ excitation: holon
- Problem: no coherent quasiparticle with electron quantum numbers
Electron fractionalization and holons

- Take out an electron
- Spin and charge degrees move independently
- Hole fractionalized into spinon and gauge-charged, spinless charge $+e$ excitation: holon
- Problem: no coherent quasiparticle with electron quantum numbers
Electron fractionalization and holons

- Take out an electron
- Spin and charge degrees move independently
- Hole fractionalized into spinon and gauge-charged, spinless charge $+e$ excitation: holon
- Problem: no coherent quasiparticle with electron quantum numbers

$\frac{1}{\sqrt{2}} \left[ \begin{array}{c} \text{Spin} \\ \text{Charge} \end{array} \right] = \begin{array}{c} \text{Spinon} \\ \text{Holon} \end{array}$

Slides online: andrea-allais.rhcloud.com
Binding of holon and spinon

- Attractive spinon-holon interaction naturally present in the model
- Holon and spinon form a bound state

\[ \frac{1}{\sqrt{2}} \left[ \begin{array}{c} \vdots \\ \end{array} - \begin{array}{c} \vdots \\ \end{array} \right] \]


Slides online: andrea-allais.rhcloud.com
Binding of holon and spinon

Attractive spinon-holon interaction naturally present in the model
Holon and spinon form a bound state

\[ \frac{1}{\sqrt{2}} \left[ \begin{array}{c} \uparrow \downarrow \\ \downarrow \uparrow \end{array} \right] \]


Slides online: andrea-allais.rhcloud.com
Fractionalized Fermi liquid (FL*)

\[ \begin{align*}
\left. \begin{array}{c}
\text{Emergent gauge field} \\
\text{Gauge-neutral, spin 1/2, charge} \\
+e \text{ fermions} \\
\text{Distinguishing feature:} \\
\text{non-trivial Luttinger count}
\end{array} \right. \\
\end{align*} \]

\[ \begin{align*}
\mathbf{a} &= \frac{1}{\sqrt{2}} \left[ \begin{array}{c}
\mathbf{A} \\
\mathbf{B} \\
\mathbf{C} \\
\mathbf{D}
\end{array} \right] \\
\mathbf{a'} &= \frac{1}{\sqrt{2}} \left[ \begin{array}{c}
\mathbf{E} \\
\mathbf{F} \\
\mathbf{G} \\
\mathbf{H}
\end{array} \right]
\end{align*} \]


Slides online: andrea-allais.rhcloud.com
Luttinger count

For a regular Fermi liquid of spin 1/2 particles:

\[ 2 \frac{V_{\text{Fermi surface}}}{V_{\text{Brillouin zone}}} = N_{\text{fermions per unit cell}} \mod 2 \]


Slides online: andrea-allais.rhcloud.com
Luttinger count

For a regular Fermi liquid of spin 1/2 particles:

\[ 2 \frac{V_{\text{Fermi surface}}}{V_{\text{Brillouin zone}}} = N_{\text{fermions per unit cell}} \mod 2 \]

Proof:

- Put system on a torus
- Thread magnetic flux through the torus
- Track momenta of low energy excitations


Slides online: andrea-allais.rhcloud.com
Luttinger count

- For a regular Fermi liquid of spin 1/2 particles:

\[ 2 \frac{V_{\text{Fermi surface}}}{V_{\text{Brillouin zone}}} = N_{\text{fermions per unit cell}} \mod 2 \]

- Proof:
  - Put system on a torus
  - Thread magnetic flux through the torus
  - Track momenta of low energy excitations

- FL* has deconfined emergent gauge degrees of freedom
  - Extra low energy excitations on a torus
  - Fermi surface area \( p \) although \( 1 + p \) holes per unit cell


Slides online: andrea-allais.rhcloud.com
Can we cook up a model hamiltonian for this physics?
Valence bonds configurations have non-zero overlap
\[ \langle \begin{array}{c} \varepsilon \varepsilon \varepsilon \varepsilon \\ \varepsilon \varepsilon \varepsilon \varepsilon \end{array} | \begin{array}{c} \varepsilon \varepsilon \varepsilon \varepsilon \\ \varepsilon \varepsilon \varepsilon \varepsilon \end{array} \rangle \neq 0 \]
Let’s ignore that (or perform similarity transformation)
Hilbert space spanned by dimer coverings
Simplest hamiltonian in this space:
\[ H_{RK} = \sum [ -J | \begin{array}{c} \varepsilon \\ \varepsilon \end{array} \rangle \langle \begin{array}{c} \varepsilon \\ \varepsilon \end{array} | + V | \begin{array}{c} \varepsilon \\ \varepsilon \end{array} \rangle \langle \begin{array}{c} \varepsilon \\ \varepsilon \end{array} | ] \]


Slides online: andrea-allais.rhcloud.com
RK point

\[ H_{\text{RK}} = \sum \left[ -J \left| \begin{array}{c} \uparrow \downarrow \end{array} \right \rangle \left\langle \downarrow \uparrow \right| + V \left| \begin{array}{c} \downarrow \downarrow \end{array} \right \rangle \left\langle \uparrow \uparrow \right| \right] \]

- Liquid phase at critical point \( V = J \) (RK point):
  - Ground state is equal weight superposition of dimer configurations
- Surrounded by crystalline phases


Slides online: andrea-allais.rhcloud.com
Topological order at the RK point

- Bipartite (red and black) lattice
Topological order at the RK point

- Bipartite (red and black) lattice
- Arrow red to black on a reference configuration
Topological order at the RK point

- Bipartite (red and black) lattice
- Arrow red to black on a reference configuration
- Arrow black to red on a given configuration
Topological order at the RK point

- Bipartite (red and black) lattice
- Arrow red to black on a reference configuration
- Arrow black to red on a given configuration
- Directed, non-intersecting loops

Slides online: andrea-allais.rhcloud.com
Topological order at the RK point

- On torus, winding numbers are constants of the motion
- They label degenerate ground states
- Degeneracy $\mathbb{Z} \times \mathbb{Z}$: same as deconfined compact U(1) gauge theory

Slides online: andrea-allais.rhcloud.com
Hamiltonian for U(1) gauge theory

\[ H = \sum_x \left[ k_1 E^2 + k_2 (\nabla \times E)^2 - \gamma \cos \nabla \times A \right], \quad \nabla \cdot E = 0 \]

- Critical point (and field theory) when \( k_1 \) changes sign
- Deconfined critical point of U(1) gauge theory
- RK point is an infinitely-multicritical point within this class
Dimer-doped dimer model

- Add fermionic, spin-carrying dimers

\[ H = H_{RK} + \sum \left[ -t_1 |\overline{11}\rangle \langle 11| - t_2 |\overline{11}\rangle \langle 11| + \cdots \right] \]

- This perturbation leads away from the deconfined critical point
- However, at intermediate scales, still expect no order
- Eventually, at scale set by confinement length, system finds crystalline order


Slides online: andrea-allais.rhcloud.com
Exact diagonalization results

- Dispersion for single fermionic dimer
- Parameters determined by connecting to the $t$-$J$ model
- Many dimers would fill pockets
- Even single particle has non-trivial residue $Z(k)$
- Back side of the pocket couples very weakly to the electron


Slides online: andrea-allais.rhcloud.com
Exact diagonalization results

- Many dimers would fill pockets
- Even single particle has non-trivial residue $Z(\mathbf{k})$
- Back side of the pocket couples very weakly to the electron


Slides online: andrea-allais.rhcloud.com
Instabilities

- At low temperature, system unstable to confinement and ordering
- In a more phenomenological model (PRB 90, 245136)
  - pairing in particle-hole channel
  - BDW with same signatures as experiment
- Superconducting instability
  - Probably need to enlarge the Hilbert space


Slides online: andrea-allais.rhcloud.com
Future developments

- Variational approach
  - Finite density
  - Spectral function
  - Ordering instabilities (SC, BDW)
- Connection to DMFT
  - Dimer states are optimal for two-sites cluster DMFT (PRB 80 064501)
- More realistic models
  - Deconfined critical point at Néel-VBS transition
  - $J$-$Q$ Model (PRL 98 227202)

Ferrero, Cornaglia, De Leo, Parcollet, Kotliar, Georges, Phys. Rev. B 80, 064501 (2009)

Sandvik, Phys. Rev. Lett. 98 227202

Slides online: andrea-allais.rhcloud.com