A quantum dimer model for the pseudogap metal

Andrea Allais

Harvard University Physics Department

Aspen Center for Physics - February 20, 2015

Slides online: andrea-allais.rhcloud.com

Goal:

- describe a simple model
- hosting a fractionalized Fermi liquid phase
- possibly realized in the pseudogap phase of cuprates

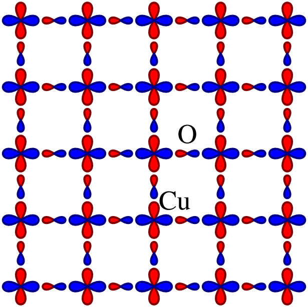
Collaborators

Matthias Punk
(Innsbruck)

Debanjan Chowdhury
(Harvard)

Subir Sachdev
(Harvard)

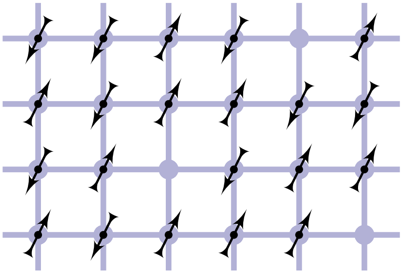
Cuprates



- Layered, quasi-2d materials
- Layers: decorated copper-oxygen square lattice
- Most interesting physics: less than 1 electron per site.
- Well described by Hubbard model

Hubbard model

$$H = \sum_{\boldsymbol{r}} \left[\sum_{\boldsymbol{a}} -t_a c^{\dagger}_{\boldsymbol{r}+\boldsymbol{a}} c_{\boldsymbol{r}} + \frac{1}{2} U n_{\boldsymbol{r}} (n_{\boldsymbol{r}} - 1) \right]$$



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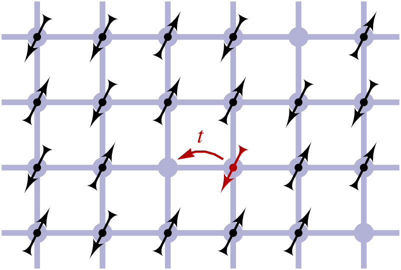

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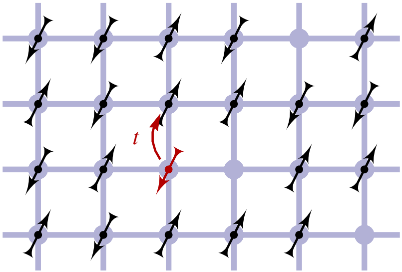

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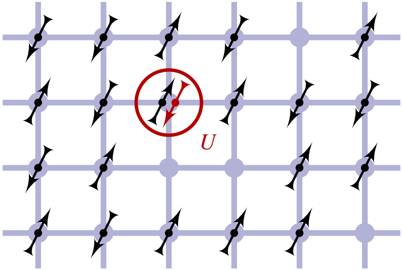
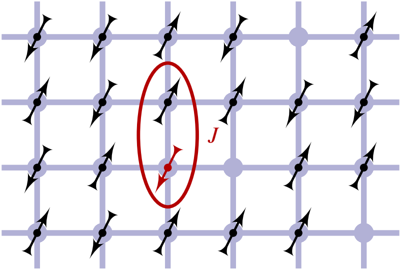
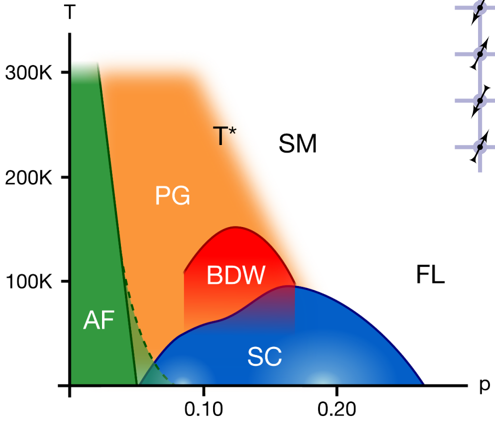


t-J model

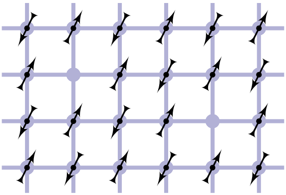
$$H = \sum_{\boldsymbol{r},\boldsymbol{a}} \left[-t_{\boldsymbol{a}} c^\dagger_{\boldsymbol{r}+\boldsymbol{a}} c_{\boldsymbol{r}} + \frac{1}{2} J_{\boldsymbol{a}} \boldsymbol{S}_{\boldsymbol{r}} \cdot \boldsymbol{S}_{\boldsymbol{r}+\boldsymbol{a}} \right]$$



Cuprates phase diagram



Antiferromagnet (AF):



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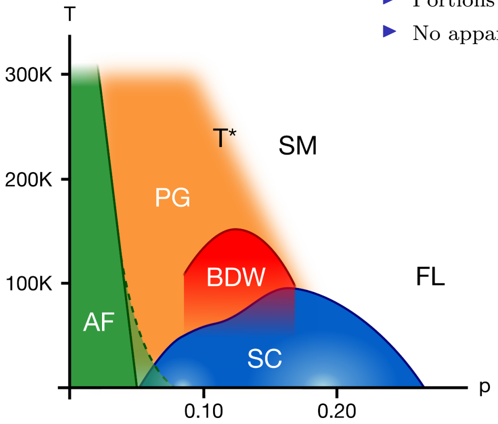
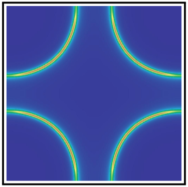


Pseudogap (PG):

- Portions of the Fermi surface are gapped
- No apparent order



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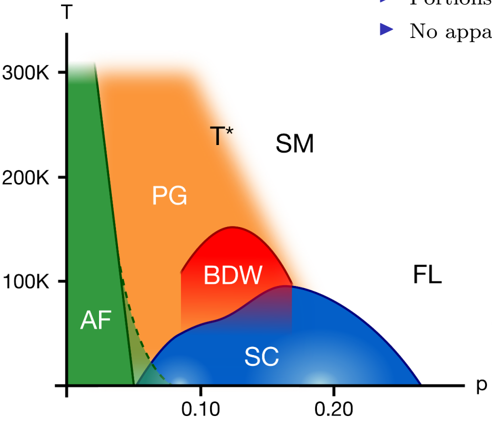

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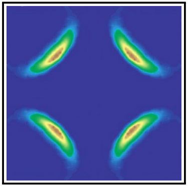
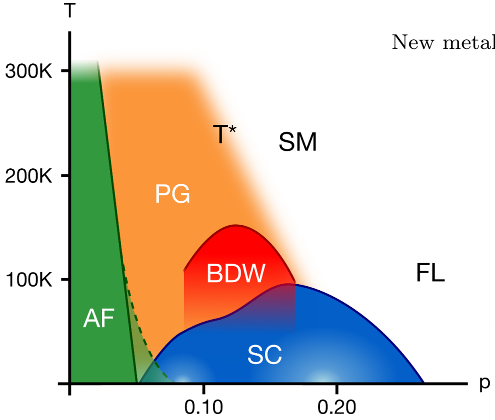


K. Shen *et al.*

Science **307**, 901 (2005)

Cuprates phase diagram

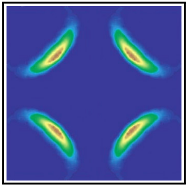


Pseudogap (PG):

Thermal fluctuations of low temperature orders

OR

New metallic state with topological order



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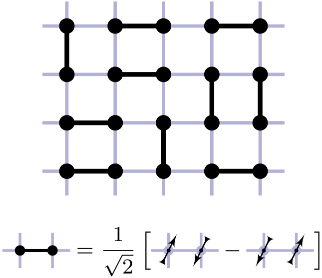
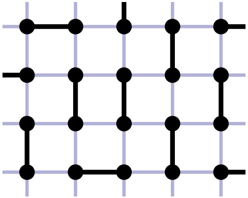
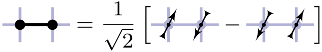
Spin liquid



- Paramagnetic state with no long range order
- Emergent gauge field
- Liquid state: correlations decay fast
- Pictorially: superposition of many spin singlet pair configurations

Spin liquid





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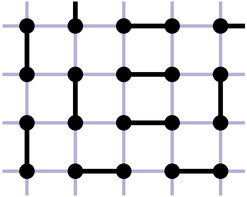

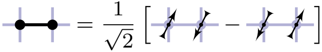

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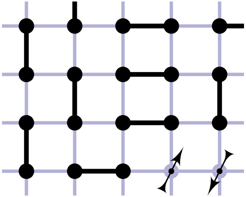
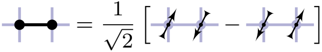
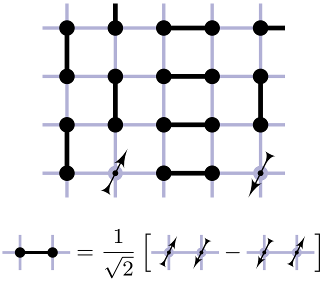
Spinons





- Create triplet excitation
- Pay energy cost

Spinons



- Create triplet excitation
- Pay energy cost
- Spins can wander away at little to no extra cost

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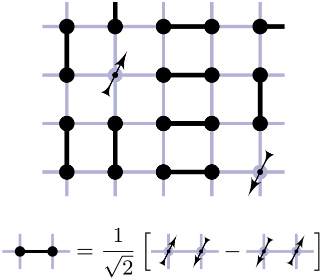

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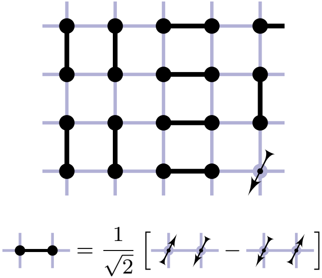

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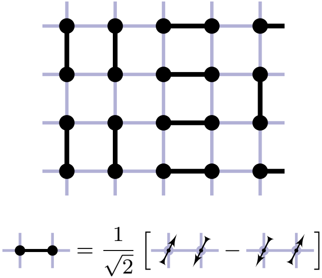
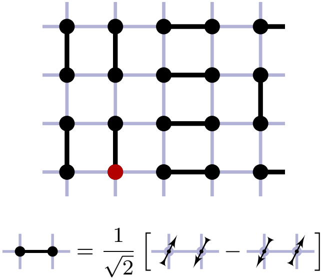


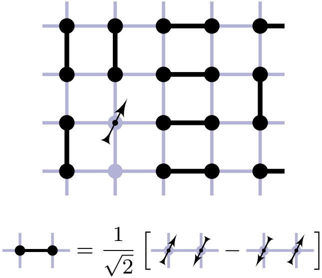
- Create triplet excitation
- Pay energy cost
- Spins can wander away at little to no extra cost
- Neutral, spin 1/2 excitation, gauge-charged: spinon

Electron fractionalization and holons



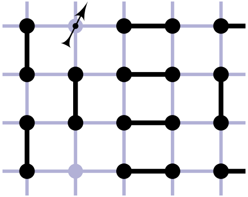
- Take out an electron

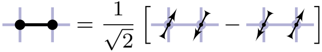
Electron fractionalization and holons



- Take out an electron
- Spin and charge degrees move independently

Electron fractionalization and holons





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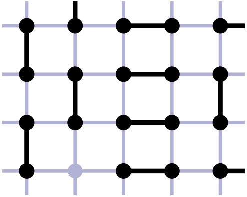

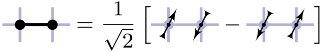

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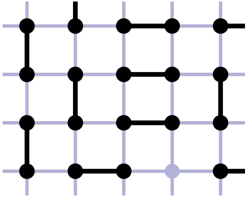

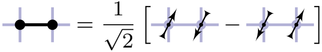

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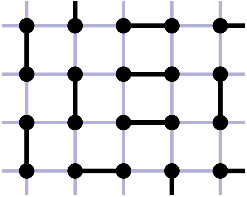

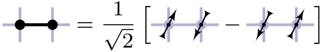

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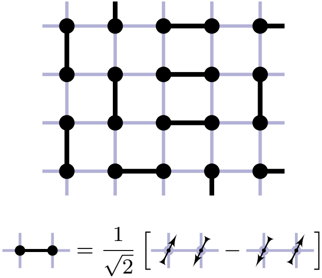

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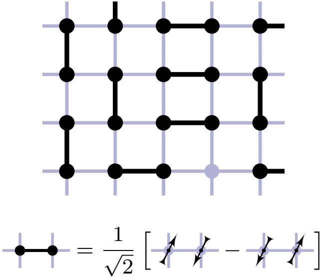

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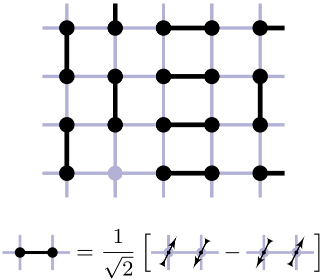

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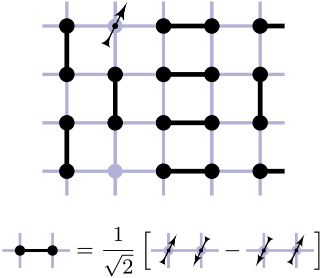

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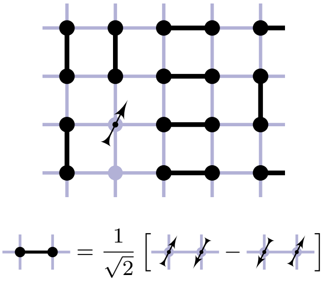

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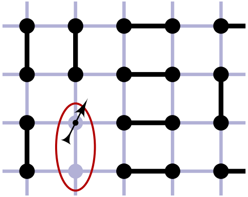
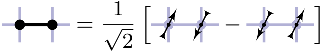
Binding of holon and spinon





- Attractive spinon-holon interaction naturally present in the model
- Holon and spinon form a bound state

Kaul, Kolezhuk, Levin, Sachdev, Senthil, Phys. Rev. B **75**, 235122 (2007)

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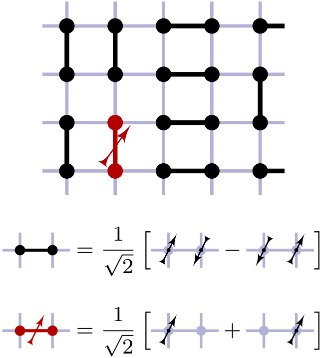

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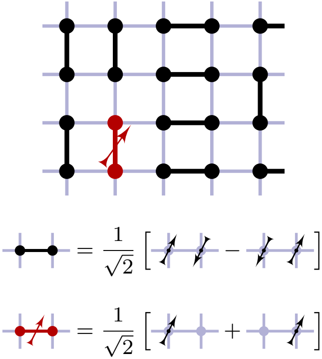
Fractionalized Fermi liquid (FL*)



- Emergent gauge field
- Gauge-neutral, spin 1/2, charge $+e$ fermions
- Distinguishing feature: non-trivial Luttinger count

Kaul, Kolezhuk, Levin, Sachdev, Senthil, Phys. Rev. B **75**, 235122 (2007)

Punk, Allais, Sachdev, arXiv:1501.00978 (2015)

Luttinger count

- For a regular Fermi liquid of spin 1/2 particles:

$$2\,\frac{V_{\text{Fermi surface}}}{V_{\text{Brilllouin zone}}} = N_{\text{fermions per unit cell}} \bmod 2$$

Oshikawa, Phys. Rev. Lett. **84**, 3370 (2000)

Senthil, Sachdev, Vojta, Phys. Rev. Lett. **90**, 216403 (2003)

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- Proof:
  - Put system on a torus
  - Thread magnetic flux through the torus
  - Track momenta of low energy excitations

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- Proof:
  - Put system on a torus
  - Thread magnetic flux through the torus
  - Track momenta of low energy excitations

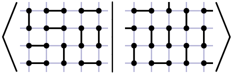
- FL* has deconfined emergent gauge degrees of freedom
  - Extra low energy excitations on a torus
  - Fermi surface area p although $1+p$ holes per unit cell

Oshikawa, Phys. Rev. Lett. **84**, 3370 (2000)

Senthil, Sachdev, Vojta, Phys. Rev. Lett. **90**, 216403 (2003)

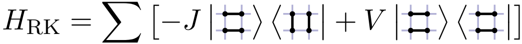
Quantum dimer model

- Can we cook up a model hamiltonian for this physics?
- Valence bonds configurations have non-zero overlap



$$\neq 0$$

- Let's ignore that (or perform similarity transformation)
- Hilbert space spanned by dimer coverings
- Simplest hamiltonian in this space:

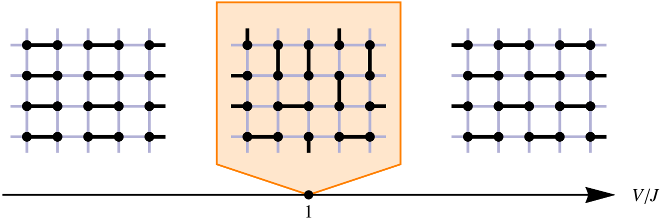


Rokhsar, Kivelson, Phys. Rev. Lett. **61**, 2376 (1988)

RK point

$$H_{\mathrm{RK}} = \sum \left[-J \left| = \right\rangle \left\langle \| \right| + V \left| = \right\rangle \left\langle = \right| \right]$$

- Liquid phase at critical point $V = J$ (RK point):
  - Ground state is equal weight superposition of dimer configurations
- Surrounded by cristalline phases



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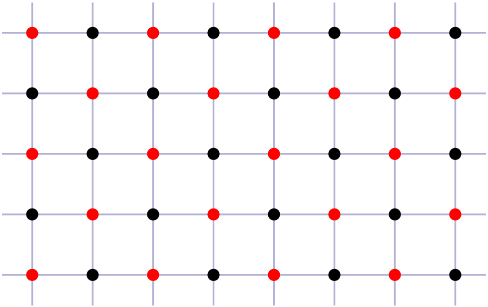
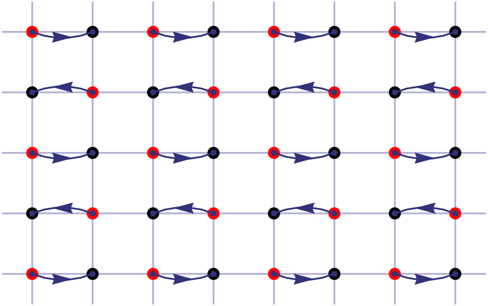
Topological order at the RK point

- Bipartite (red and black) lattice



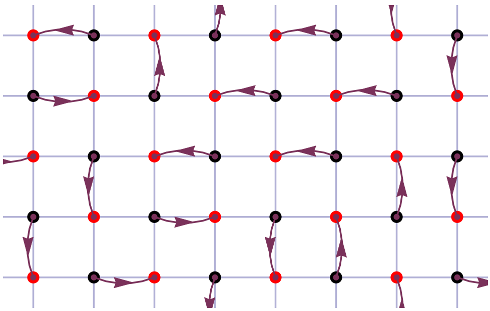
Topological order at the RK point

- Bipartite (red and black) lattice
- Arrow red to black on a reference configuration



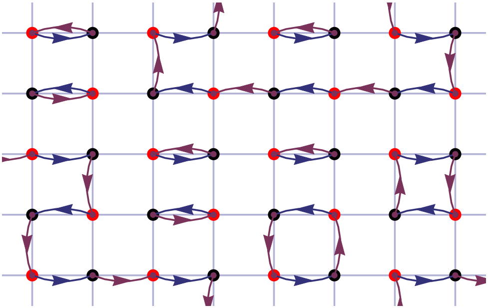
Topological order at the RK point

- Bipartite (red and black) lattice
- Arrow red to black on a reference configuration
- Arrow black to red on a given configuration



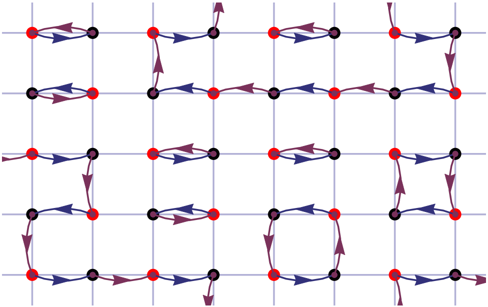
Topological order at the RK point

- Bipartite (red and black) lattice
- Arrow red to black on a reference configuration
- Arrow black to red on a given configuration
- Directed, non-intersecting loops



Topological order at the RK point

- On torus, winding numbers are constants of the motion
- They label degenerate ground states
- Degeneracy $\mathbb{Z} \times \mathbb{Z}$: same as deconfined compact U(1) gauge theory



Field theory

- Hamiltonian for U(1) gauge theory

$$H = \sum_{\boldsymbol{x}} \left[k_1 \boldsymbol{E}^2 + k_2 (\nabla \times \boldsymbol{E})^2 - \gamma \cos \nabla \times \boldsymbol{A} \right] , \qquad \nabla \cdot \boldsymbol{E} = 0$$

- Critical point (and field theory) when k_1 changes sign
- Deconfined critical point of U(1) gauge theory
- RK point is an infinitely-multicritical point within this class

Moessner, Sondhi, Fradkin, Phys. Rev. B **65**, 024504 (2001)

Vishwanath, Balents, Senthil, Phys. Rev. B **69**, 224416 (2004)

Dimer-doped dimer model

- Add fermionic, spin-carrying dimers

$H = H_{\mathrm{RK}} + \sum \big[-t_1 \,\big|$  $\big\rangle \big\langle$  $\big| - t_2 \,\big|$  $\big\rangle \big\langle$  $\big| + \cdots \big]$

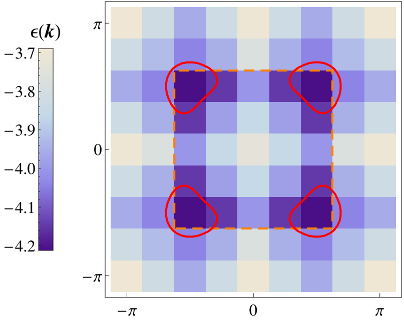
- This perturbation leads away from the deconfined critical point
- However, at intermediate scales, still expect no order
- Eventually, at scale set by confinement length, system finds cristalline order

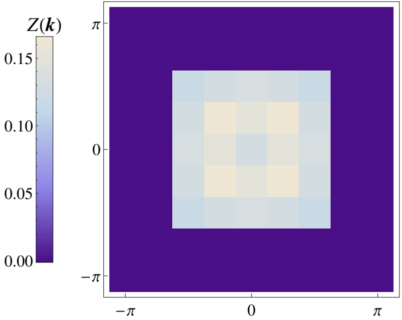
Punk, Allais, Sachdev, arXiv:1501.00978 (2015)

Exact diagonalization results

- Dispersion for single fermionic dimer
- Parameteres determined by connecting to the t-J model

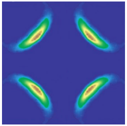
- Many dimers would fill pockets
- Even single particle has non-trivial residue $Z(\boldsymbol{k})$
- Back side of the pocket couples very weakly to the electron





Punk, Allais, Sachdev, arXiv:1501.00978 (2015)

Exact diagonalization results



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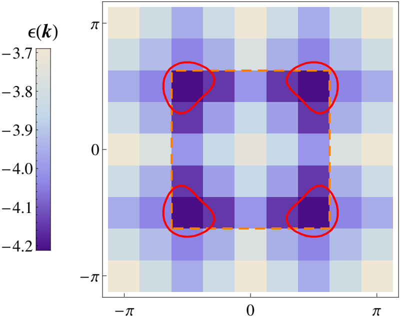

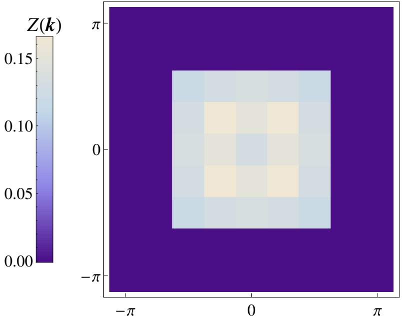

Punk, Allais, Sachdev, arXiv:1501.00978 (2015)

Instabilities

- At low temperature, system unstable to confinement and ordering
- In a more phenomenological model (PRB **90**, 245136)
  - pairing in particle-hole channel
  - BDW with same signatures as experiment
- Superconducting instability
  - Probably need to enlarge the Hilbert space

Chowdhury, Sachdev, Phys. Rev. B **90**, 245136 (2015)

Future developments

- Variational approach
  - Finite density
  - Spectral function
  - Ordering instabilities (SC, BDW)
- Connection to DMFT
  - Dimer states are optimal for two-sites cluster DMFT (PRB **80** 064501)
- More realistic models
  - Deconfined critical point at Néel-VBS transition
  - J-Q Model (PRL **98** 227202)

Ferrero, Cornaglia, De Leo, Parcollet, Kotliar, Georges, Phys. Rev. B **80**, 064501 (2009)

Sandvik, Phys. Rev. Lett. **98** 227202