Continuity and resurgence in QFT

Aleksey Cherman
FTPI, University of Minnesota

based on work with assorted linear combinations of
Gokce Basar (Maryland U.)
Daniele Dorigoni (DAMTP, Cambridge U.),
Gerald Dunne (Connecticut U.),
and Mithat Unsal (North Carolina State U.)

arXiv:1308.0127, 1403.1277, 1410.0388, + works in progress

Argyres + Unsal 1206.1890; Dunne + Unsal 1210.2423; …

Resurgence and quantum field theory

Gerald introduced the notion of a ‘resurgent transseries’

$$\mathcal{O}(\lambda) \simeq \sum_n p_n \lambda^n + \sum_c e^{-\frac{S_c}{\lambda}} \sum_k p_{k,c} \lambda^k + \cdots$$

where the series are asymptotic, but $\mathcal{O}(\lambda)$ remains well-defined thanks to devious conspiracies between terms

This talk: how resurgence works in asymptotically-free QFTs

Involves many questions not present in QM!

Resurgence and quantum field theory

Involves many questions not present in QM!

(1) Couplings usually run, so which ‘λ’ do we mean?

(2) What can we say about p_n for large n?

$$\text{Does } p_n \sim n!\left(\frac{1}{a}\right)^n \text{ for some } a?$$

How does a depend on the number of ‘colors’ N?

QM analogy might suggest a ~ (instanton action), so that $a \sim N^1$

But some QFTs have no instantons in the first place…

and there’s evidence that $a \sim N^0$. So do we need fractional instantons?

Resurgence and quantum field theory

Involves many questions not present in QM!

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(2) What can we say about p_n for large n?

(3) What are the relevant non-perturbative (NP) saddles?

Resurgence and quantum field theory

Involves many questions not present in QM!

(1) Couplings usually run, so which 'λ' do we mean?

(2) What can we say about p_n for large n?

(3) What are the relevant non-perturbative (NP) saddles?

(4) How to do reliable semiclassical calculations of NP phenomena?

Usually the NP physics happens in a strongly-coupled domain…

Turns out that all of these issues are related.

Outline

Review what is believed — and what is known — about large-order behavior of perturbative series in QCD-like QFTs

Explain the notion of adiabatic compactification as a tool to systematically study NP physics of asymptotically-free QFTs

Renormalon ambiguities, and the mass gap, are tied to the presence of ‘fracton’ saddle points — fractional instantons.

Works even in theories without topological instantons

Resurgent structure in QFTs is realized via cancellation of ambiguities in resummation of perturbative series against ambiguities in multi-fracton amplitudes

Perturbation theory at large order on R^d

‘t Hooft, 1979

$$\mathcal{O}(\lambda) \simeq \sum_n p_n \lambda^n + \cdots$$

Widespread belief: in asymptotically-free QFTs with a mass gap Δ, with one-loop ’t Hooft-coupling beta function β_0

$$p_n \to n! \left(\frac{\beta_0}{8\pi^2}\right)^n, n \gg 1$$

Reminder: $\beta_0 = 11/3$ in SU(N) YM

Called an IR renormalon divergence, and drives appearance of apparent resummation ambiguities of order

$$\pm i\mu\, e^{-\frac{8\pi^2}{\lambda(\mu)\beta_0}} \sim \pm i\Lambda \sim \pm i\Delta$$

If you buy this, it suggests that renormalons are related to confinement and mass gaps

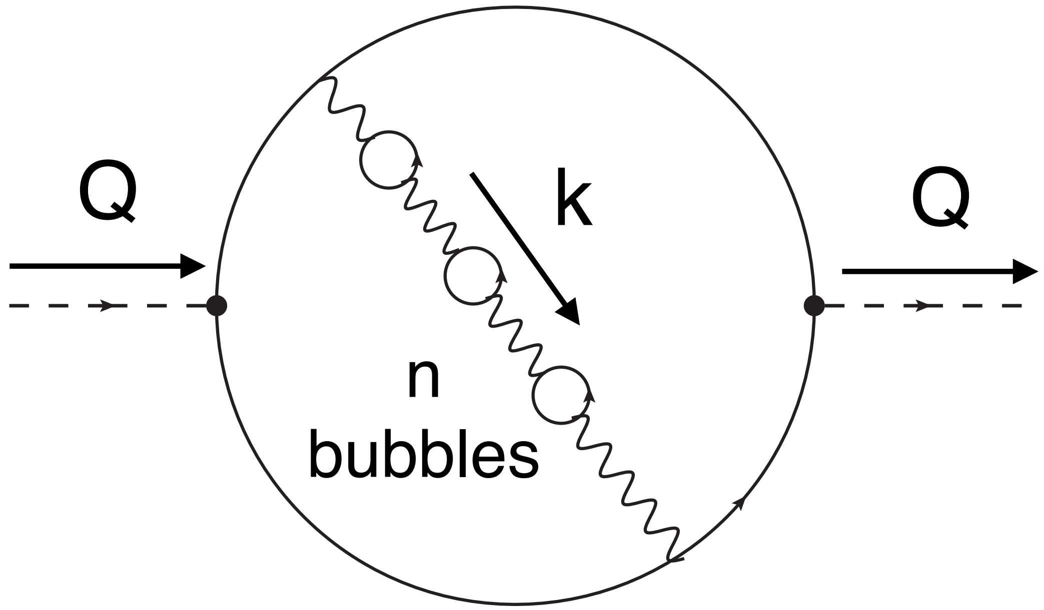
Clay Prize

But why do people believe this?

Common heuristic argument for renormalons

Review by Beneke 1998

Inspired analysis of inspirational subset of high-order diagrams for QCD



If $N_f \gg N_c$, these are the dominant diagrams.

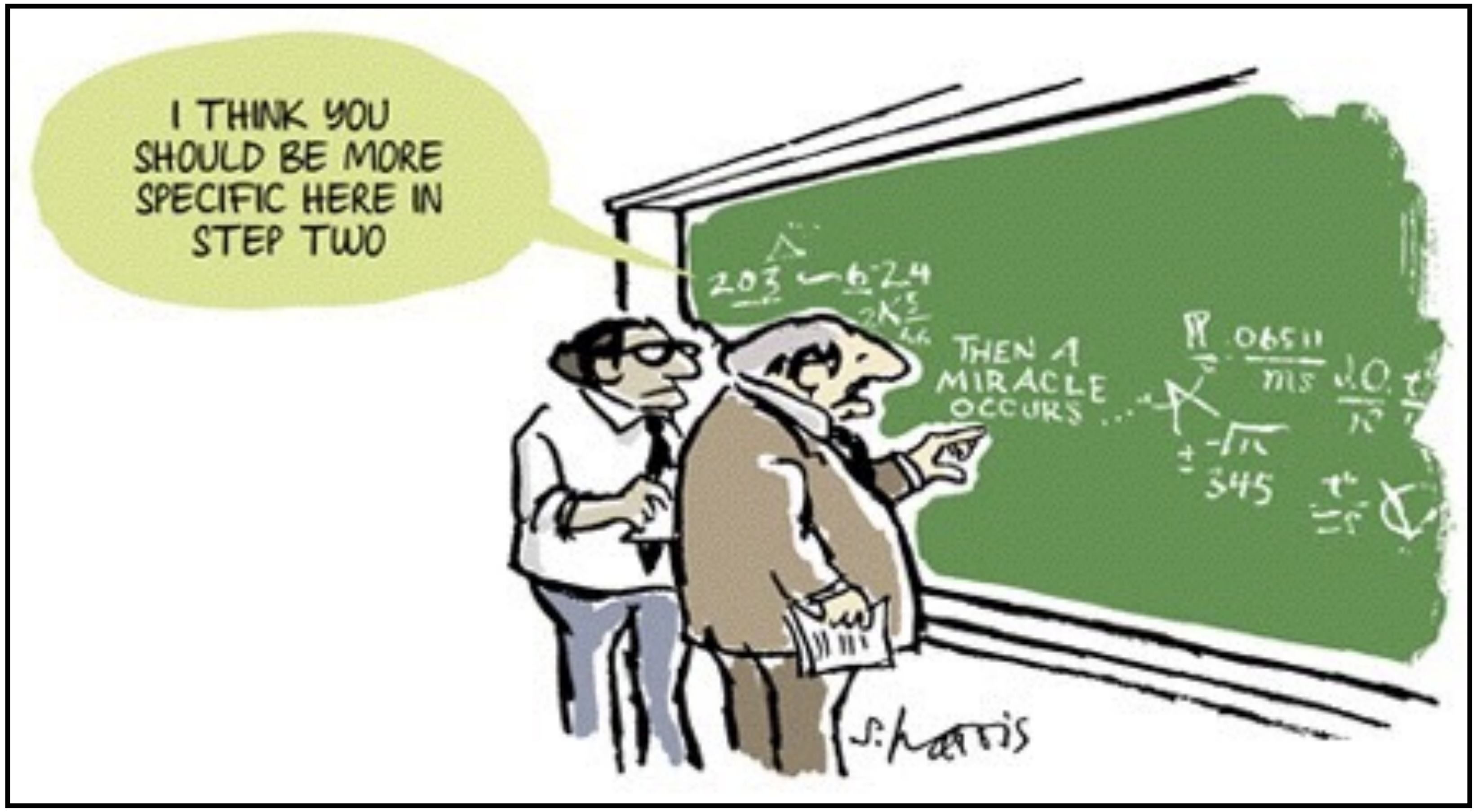
$$p_n \to n! \left(\frac{\beta_{0,f}}{8\pi^2} \right)^n, n \gg 1 \qquad \beta_{0,f} = -4/3 N_f$$

Guess that $1/N_f$ corrections conspire to give

$$\beta_{0,f} \to \beta_0 = \frac{11}{3} - \frac{4}{3} N_f$$

… and argue that it should be right even when $N_f = 0$, for YM theory

Common heuristic argument for renormalons



You might not find the historical argument entirely persuasive.

Indeed, it can give misleading results!

Dunne, Shifman, Unsal, in progress

But the result that renormalons exist, in the sense that

$$p_n \to n!\, a^{-n}, a \sim N_c^0$$

$$\text{ambiguity: } \pm i\Lambda^c \sim \pm i\Delta^c,\ c \sim N^0$$

appears to be correct.

Systematic evidence for renormalon divergences

(1) Exact solutions of some 2D asymptotically free QFTs — non-linear sigma models on R^2 — show renormalons.

Integrability or large N techniques give exact expressions for observables

Expanding in λ, large order behavior turns out to be

$$p_n \to n!\, a^{-n}, a \sim N_c^0$$

in every model examined so far.

PCM:
Kazakov, Fateev,
Wiegmann, 1990s

O(N), in context of OPEs
F. David, 1980s; Novikov, Shifman,
Vainshtein, Zakharov 1980s;

O(N) and resurgence,
Basar, AC, Dunne,
Dorigoni, Unsal, to appear

(2) Analyses using adiabatic compactifications of 4D gauge theories and 2D sigma models again implies renormalons.

Explaining this statement is the rest of the talk.

Making asymptotically-free QFTs calculable

Asymptotically free QFTs are typically strongly coupled at low energies

Under what conditions could they become weakly-coupled?

If there are scalar fields around, could use the Higgs mechanism

Then if the VEV is large compared to Λ, IR becomes weakly-coupled

Of course, this is why the electroweak part of SM is calculable

Also an important feature in analysis of SUSY gauge theories

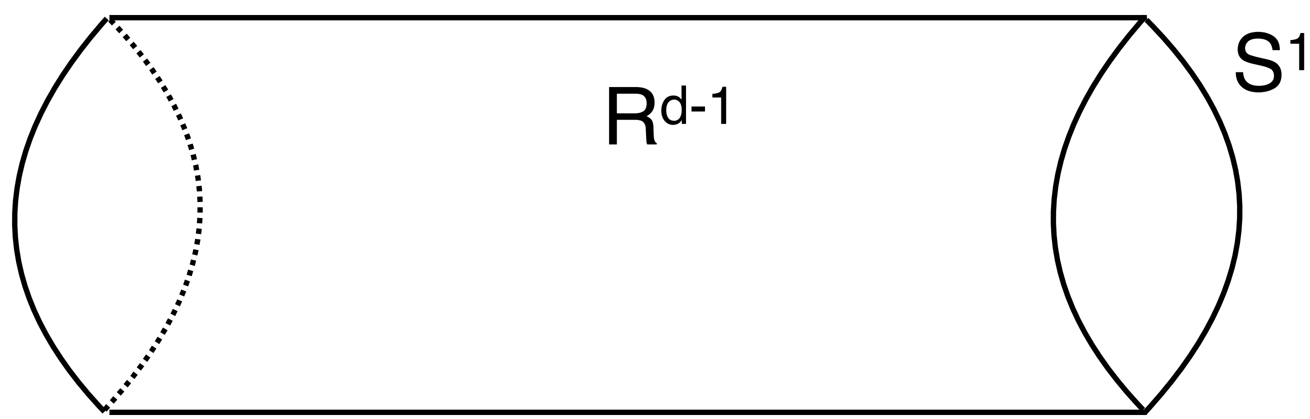
But we want to study QFTs, like QCD,
which don't include such scalar fields.

What else could one do?

Control via compactification?

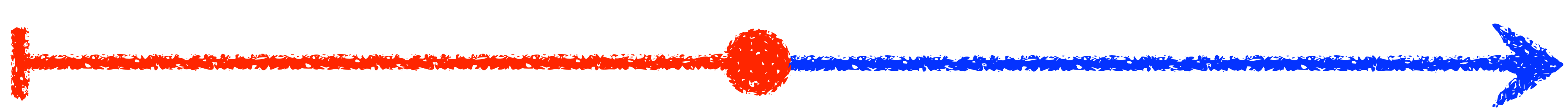
Another way to get control over an asymptotically-free QFT with a strong scale Λ on R^d is to compactify it to $R^{d-1} \times S^1$

Benefit: when S^1 size $\beta \ll \Lambda$, theory becomes weakly-coupled



With thermal BCs, $\beta = 1/T$

Heavy cost #1: often the small-volume theory is separated from large volume phase by a phase transition



	small β	large β
Matrix QFTs	$F/N^2 \sim 1$	$F/N^2 \sim 0$

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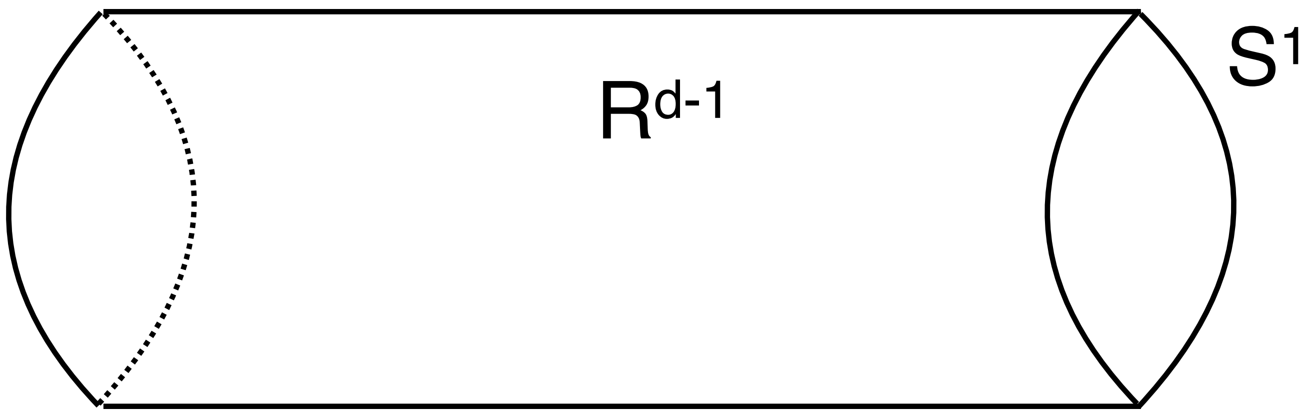


With thermal BCs, $\beta = 1/T$

Heavy cost #2: small-β system 'forgets' the characteristic scale Λ of the R^d system

	Large β	Small β
Mass gap Δ	$\Delta = \Lambda$	$\Delta = \beta^{-1} \log(\beta\Lambda)^{\#}$

In the small-β regime, the scales are set by β, rather than Λ.

NP physics of R^d system dramatically altered by small β limit

Control via adiabatic compactifications

Large volume (low T) limit of e.g. QCD-like SU(N) gauge theories characterized by unbroken center symmetry

Z_N center symmetry is always broken at high temperature

$$\langle \frac{1}{N} \mathrm{Tr}\; e^{i \oint dx_4 A_4} \rangle \equiv \langle \Omega \rangle = \begin{cases} 0, & \mathbb{Z}_N \text{ preserved} \\ 1, & \mathbb{Z}_N \text{ broken} \end{cases}$$

Are there non-thermal compactifications which keep Z_N or its analogs unbroken, for any S^1 size?

Yes!

SU(N) YM + N_F adjoint fermions on a spatial $S^1 x R^3$;

Double-trace-deformed Yang-Mills theory on $S^1 x R^3$;

CP^{N-1} model on $S^1 x R$ with Z_N twisted BCs;

SU(N) principal chiral model on $S^1 x R$ with Z_N twisted BCs;

...

Kovtun, Unsal, Yaffe 2007; Unsal, Yaffe 2008; Shifman, Unsal 2008; Dunne, Unsal 2012; AC, Dorigoni, Dunne, Unsal 2013; …

Control via adiabatic compactifications

Motivates working with non-thermal compactifications which preserve the center symmetry, or its analogs, for any S^1 size L.

Benefit: when L << Λ, theory becomes weakly-coupled!

What about the costs we saw in thermal case?

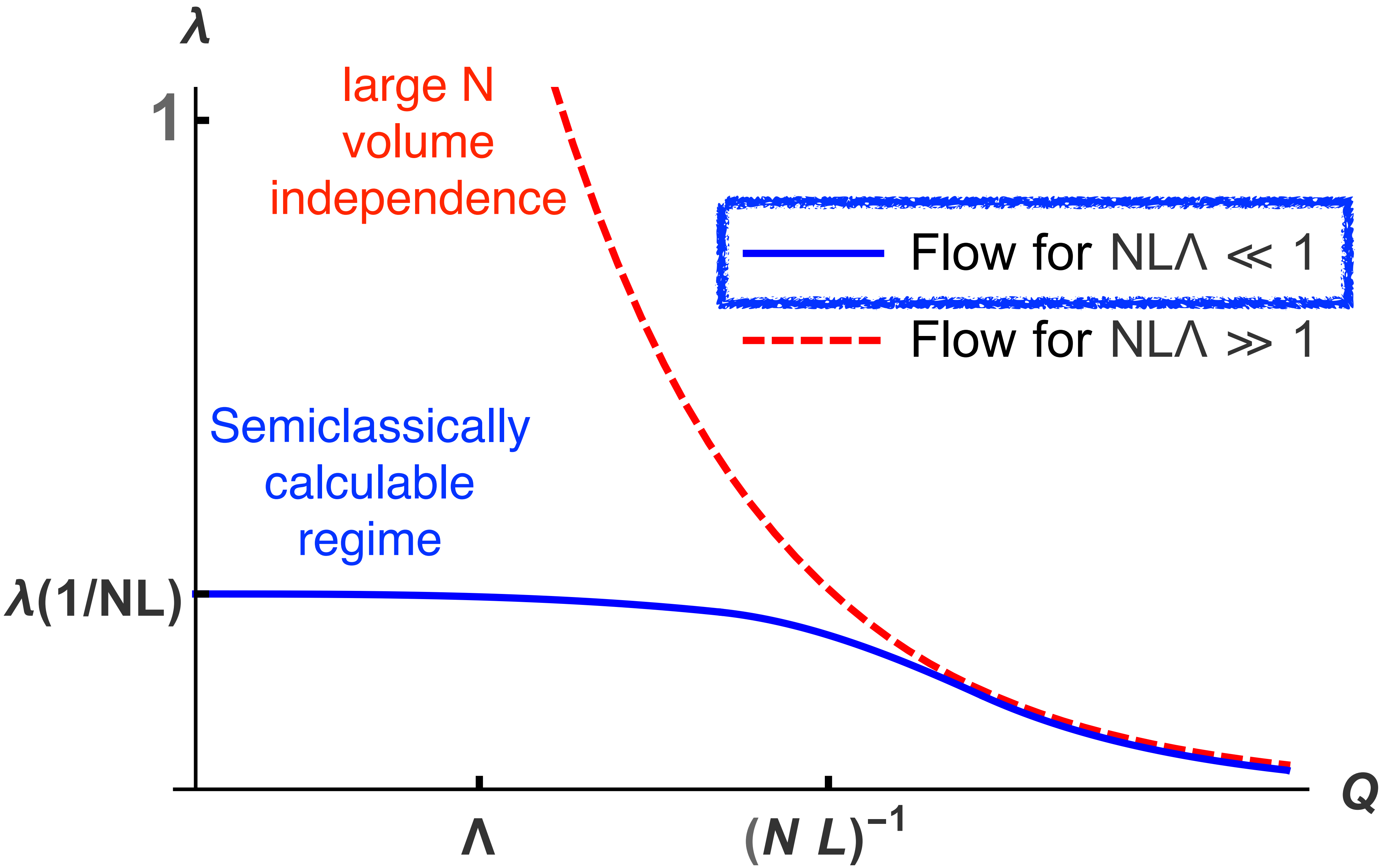
(1) By construction, no deconfinement-type phase transitions.

(2) Adiabatic small-L limit retains power law dependence on Λ.

	Large L	Small L
Mass gap Δ	$\Delta = \Lambda$	$\Delta = \Lambda\,(NL\Lambda)^{\#}$

As a result, adiabatically-compactified small L theories become an excellent test environment for resurgence theory.

Coupling flow with adiabatic compactification



$\langle \Omega \rangle = 0 \Rightarrow \langle A_4 \rangle \neq 0$, so adjoint Higgs mechanism sets in at scale $1/(NL)$.

The NLΛ << 1 regime gives a weakly-coupled theory

Physics is very rich - mass gap, renormalons present at small N L!

Worked example of resurgence in QFT

AC, Dunne, Dorigoni, Unsal 2013

Rest of the talk will discuss the 2D SU(N) Principal Chiral Model

$$S = \frac{1}{2g^2}\int_M d^2x\, \mathrm{Tr}\ \partial_\mu U \partial^\mu U^\dagger, \qquad U \in SU(N)$$

Similar discussion can be given for e.g. 2D CP^{N-1} model, or 4D gauge theories

Argyres, Unsal 2012; Dunne, Unsal 2012

PCM is interesting for two reasons. First, many similarities to QCD:

- Asymptotically free
- Matrix-like large N limit
- Dynamically generated mass gap
- Large N ‘deconfinement’ transition at high T
- Has renormalon divergences

Fateev, Kazakov, Wiegmann

Integrable, $M = R^2$ S-matrix known, so easier than QCD

Worked example of resurgence in QFT

AC, Dunne, Dorigoni, Unsal 2013

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$$S = \frac{1}{2g^2}\int_M d^2x\,\mathrm{Tr}\;\partial_\mu U \partial^\mu U^\dagger, \qquad U \in SU(N)$$

But $\pi_2[SU(N)] = 0$, so no instantons, unlike QCD!

We will see that resurgence theory puts apparent absence of NP saddles in tension with renormalon ambiguities.

Guided by resurgence, missing saddles have been found, and yield insights into NP physics behind renormalons and the mass gap

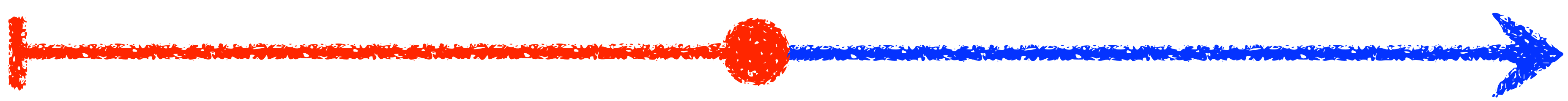
Finding a calculable regime

To explore resurgence structure of PCM, need to find a smooth weakly-coupled limit where renormalons survive.

Our approach is to put the theory on $M = R^{time} \times S^1(L)$

For small enough L, weak coupling guaranteed by asymptotic freedom

With periodic boundary conditions, S^1 would be a thermal circle!



small L	large L

	small L	large L
Free energy F	$F/N^2 \sim 1$	$F/N^2 \sim 0$

Resembles confinement/deconfinement transition in 4D YM!

In PCM, large N phase transition, finite N cross-over

Twisted boundary conditions

There is an adiabatic compactification for the PCM, but it involves twisted (non-thermal!) boundary conditions.

Consider twisted BCs using the $SU(N)_L x SU(N)_R$ symmetry

$$\begin{aligned} U(x_1, x_2 + L) &= \Omega_L U(x_1, x_2) \Omega_R^\dagger \\ &= e^{iL^{-1} H_L} U(x_1, x_2) e^{-L^{-1} H_R} \end{aligned}$$

When $L \gg \Lambda^{-1}$, choice of BCs doesn't matter thanks to the mass gap

But when $L \ll \Lambda^{-1}$, the choice matters!

Claim: unique choice of H_L, H_R such that physics appears to be adiabatically connected to large L limit

Adiabatic compactification

In general, the partition function depends on $H_{L,R}$

$$Z \to Z(L, H_L, H_R)$$

What are the desirable 'adiabaticity conditions' in terms of Z?

The large L theory has the key features

(A) A free energy scaling as $F/N^2 \sim 0$ at large N

(B) Insensitivity of theory to changes in BCs

Would like to maintain both of them, to the extent possible, at small L.

Adiabaticity conditions

In terms of $2H_{V,A} = H_L \pm H_R$ these conditions translate to

A

$$\frac{L^2}{N^2}\mathcal{F}|_{H_V,H_A} \sim 0$$

Stay in 'confining' phase

Total insensitivity to BCs is not possible at small L.
Best we can do is demand

B

$$\frac{\partial\left[\mathcal{V}^{-1}\log Z(L)\right]}{\partial H_V} = \langle J_x^V \rangle_{H_V,H_A} = 0$$

$$\frac{\partial\left[\mathcal{V}^{-1}\log Z(L)\right]}{\partial H_A} = \langle J_x^A \rangle_{H_V,H_A} = 0$$

Free energy-extremizing BCs

Our task: compute $F(L; H_A, H_V)$ at small L, where theory is weakly coupled, and look at large N scaling of extrema

Adiabatic compactification

Result of this calculation is that desired extremum is $\Omega_A = 1$, while Ω_V has Z_N center symmetric form

$$\Omega_V = e^{i\frac{\pi\nu}{N}} \begin{pmatrix} 1 & & & \\ & e^{i\frac{2\pi}{N}} & & \\ & & \ddots & \\ & & & e^{i\frac{2\pi(N-1)}{N}} \end{pmatrix}$$

v = 0,1 for N odd, even

$$\frac{1}{N^2}\mathcal{F}(L \ll \Lambda^{-1})|_{\Omega_V} = \frac{1}{N^2}\frac{\pi}{6(NL)^2}$$

Goes to zero at large N

'Confinement' even at small L!

Z_N-symmetric BCs give desired adiabatic small-volume limit, with $NL\Lambda$ as a control parameter.

Related construction of an adiabatic small L limit known for 4D gauge theories

Unsal, Yaffe; Shifman, Unsal; ...

Perturbation theory at small L

Consider dependence of ground state energy on $\lambda = g^2 N$.

On R^2, integrability calculations of Kazakov, Fateev, Wiegmann imply:

$$\sim \pm i e^{-\frac{8\pi}{g^2 N}}$$

If small-L limit is adiabatic, expect size of renormalon ambiguity to move by order-1 amount as L goes from large to small.

$$\sim \pm i e^{-\frac{\#}{g^2 N}}$$

For small L, 2D PCM describable via a 1D EFT: quantum mechanics with a Z_N-symmetric background gauge field

This will let us make concrete statements about large-order behavior…

Perturbation theory at small L

SU(2) Example

$$U = \begin{pmatrix} \cos\theta e^{i\phi_1} & i\sin\theta e^{i\phi_2} \\ i\sin\theta e^{-i\phi_2} & \cos\theta e^{-i\phi_1} \end{pmatrix}$$

Hopf parametrization

$$S = \frac{1}{g^2}\int_{\mathbb{R}\times S^1} dt\, dx\, \left[(\partial_\mu\theta)^2 + \cos^2\theta(\partial_\mu\phi_1)^2 + \sin^2\theta(\partial_\mu\phi_2 + \xi\delta_{\mu,x})^2\right]$$

KK reduction

Imprint of Z_N twisted BCs: $\xi = 2\pi/(NL) = \pi/L$

$$S = \frac{L}{g^2}\int dt\left[\dot\theta^2 + \cos^2\theta\,\dot\phi_1^2 + \sin^2\theta\,\dot\phi_2^2 + \xi^2\sin^2\theta\right]$$

Compute perturbative expansion for ground state energy:

$$\mathcal{E}(\lambda) = E\xi^{-1} = \sum_{n=0}^{\infty} p_n\lambda^n$$

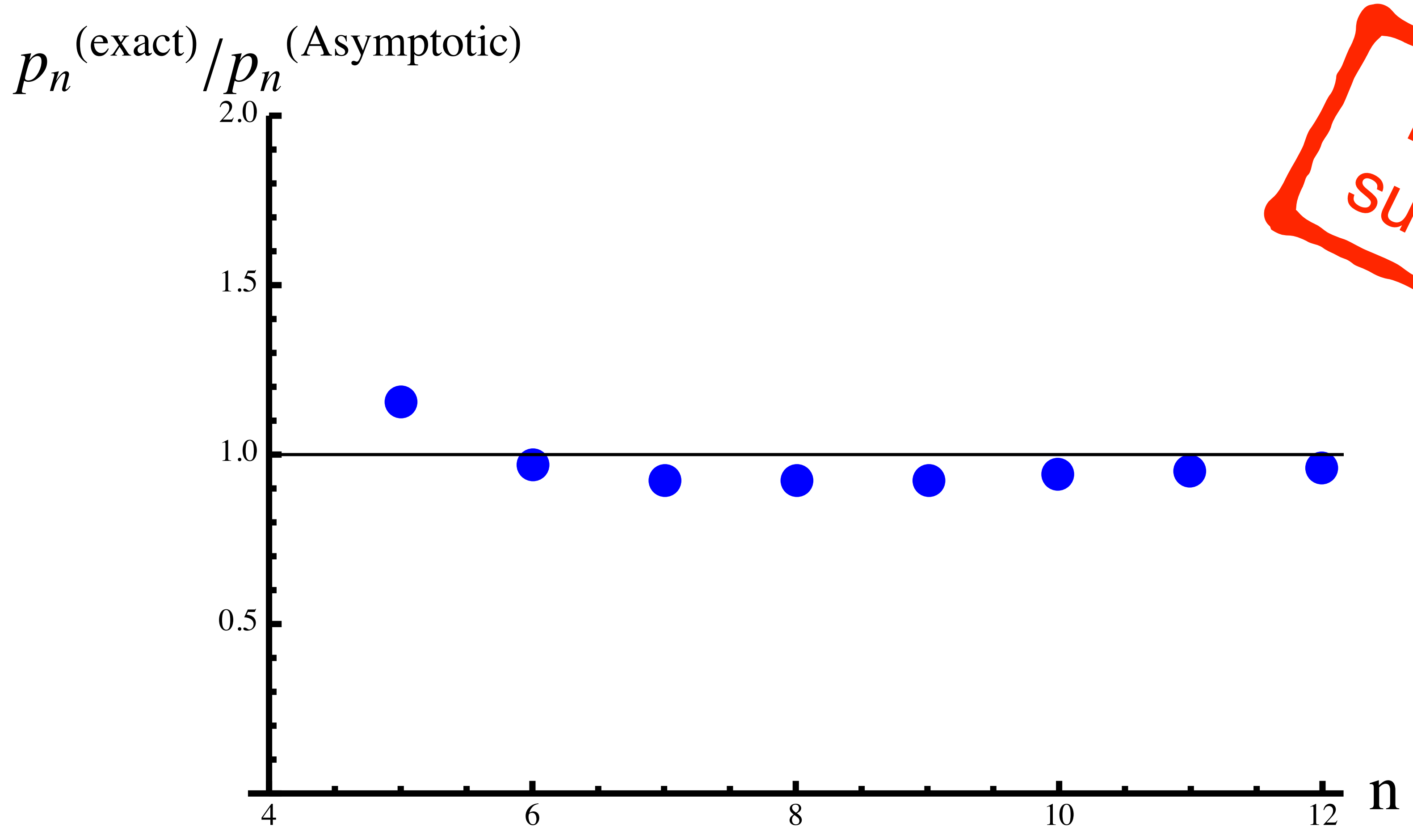
Large order structure of perturbation theory

In QM, calculating large orders in perturbation theory is doable. Result:

Stone, Reeve 1978

$$p_n \sim -\frac{2}{\pi}(16\pi)^{-n}\, n! \left[1 - \frac{5}{2n} + \mathcal{O}\left(n^{-2}\right)\right]$$

Factorially growing and non-alternating series!



Non-perturbative ambiguity

$$\mathcal{S}_{\pm}\mathcal{E}(\lambda) = \int_{C_{\pm}} dt e^{-t/g^2} B\mathcal{E}(t)$$

$$= \mathrm{Re}\mathcal{S}\mathcal{E}(\lambda) \mp i\frac{32\pi}{\lambda} e^{-16\pi/\lambda}$$

Renormalon ambiguities survive the adiabatic small L limit!

Resurgence theory: P ambiguity must cancel against NP ambiguity

$$\mathrm{Im}\left[\mathcal{S}_{\pm}\mathcal{E}(g^2) + [\mathcal{F}\bar{\mathcal{F}}]_{\pm}\right] = 0, \text{ up to } \mathcal{O}\left(e^{-4S_F}\right)$$

plus more intricate relations at higher orders

But what are the relevant NP saddle points in the PCM?

Recall $\pi_2[SU(N)] = 0$…

Uhlenbeck 1985...

Non-topological saddle points

Finite-action `uniton' solutions of PCM 2nd order EoMs are known

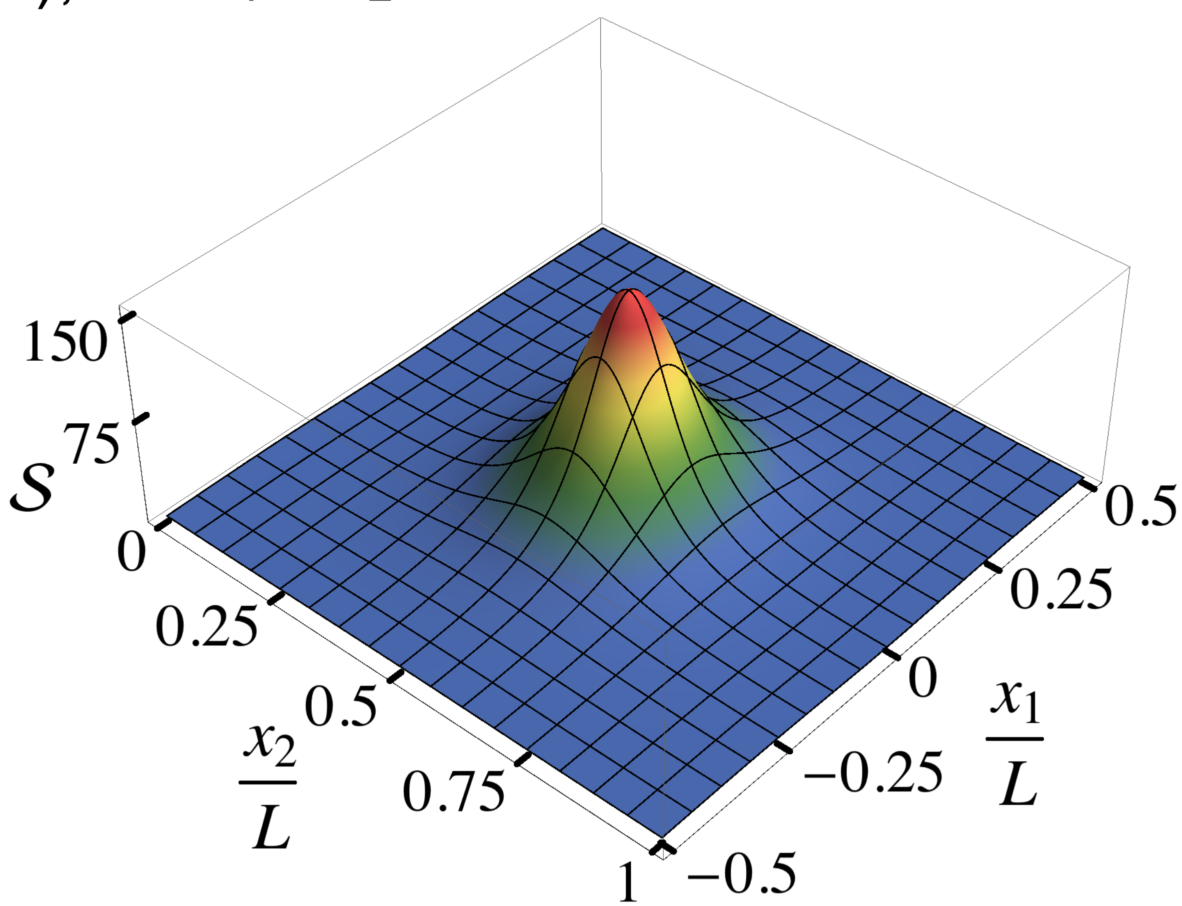
Contrast with instantons, which solve 1st order self-dual EoMs.

Construction uses observation that CP^{N-1} is a geodesic submanifold of SU(N)

CP^{N-1} instantons lift to uniton solutions in SU(N) PCM

$$U(z,\bar{z}) = e^{i\pi/N}(1-2\mathbb{P}) \qquad \mathbb{P} = \frac{v\cdot v^\dagger}{v^\dagger \cdot v}$$

v(z), $z = x_1 + i\, x_2$ is the CP^{N-1} instanton in homogeneous coordinates



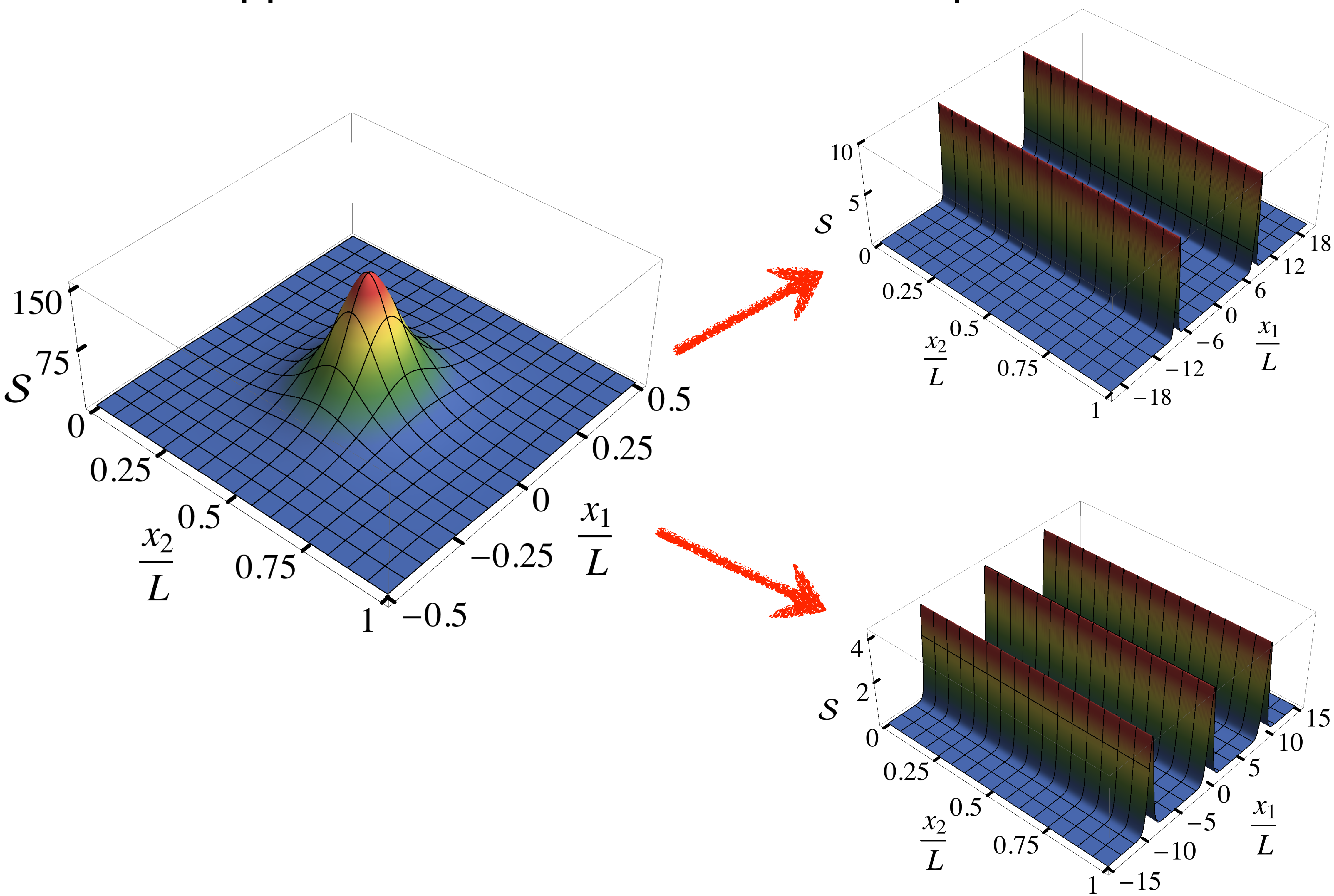
$$S_{\text{uniton}} = \frac{8\pi}{g^2}$$

see also Smilga, Shifman in Schwinger model, 1994

Fractons

AC, Dorigoni, Dunne, Unsal

Uniton appearance with Z_N-twisted BCs depends on size modulus



SU(2)

SU(3)

Unitons fractionalize into N `fracton' constituents on small S^1

Fractons

AC, Dorigoni, Dunne, Unsal 2013, 2014

Explicit solutions for SU(2):

$$\theta(t;t_0) = 2\,\mathrm{arcCot}\left[e^{-\xi(t-t_0)}\right]$$

$$\bar{\theta}(t;t_0) = \pi - 2\,\mathrm{arcCot}\left[e^{-\xi(t-t_0)}\right]$$

ϕ_1 = const

ϕ_2 = const

As usual, SU(N) solutions follow by embedding into SU(2)'s

$$S_{\mathrm{fracton}} = \frac{8\pi}{g^2 N} = \frac{S_{\mathrm{uniton}}}{N}$$

N types of minimal-action fractons in SU(N)

N-1 fractons associated to N-1 simple roots of su(N)
The other - called KK fracton - associated to 'affine root'

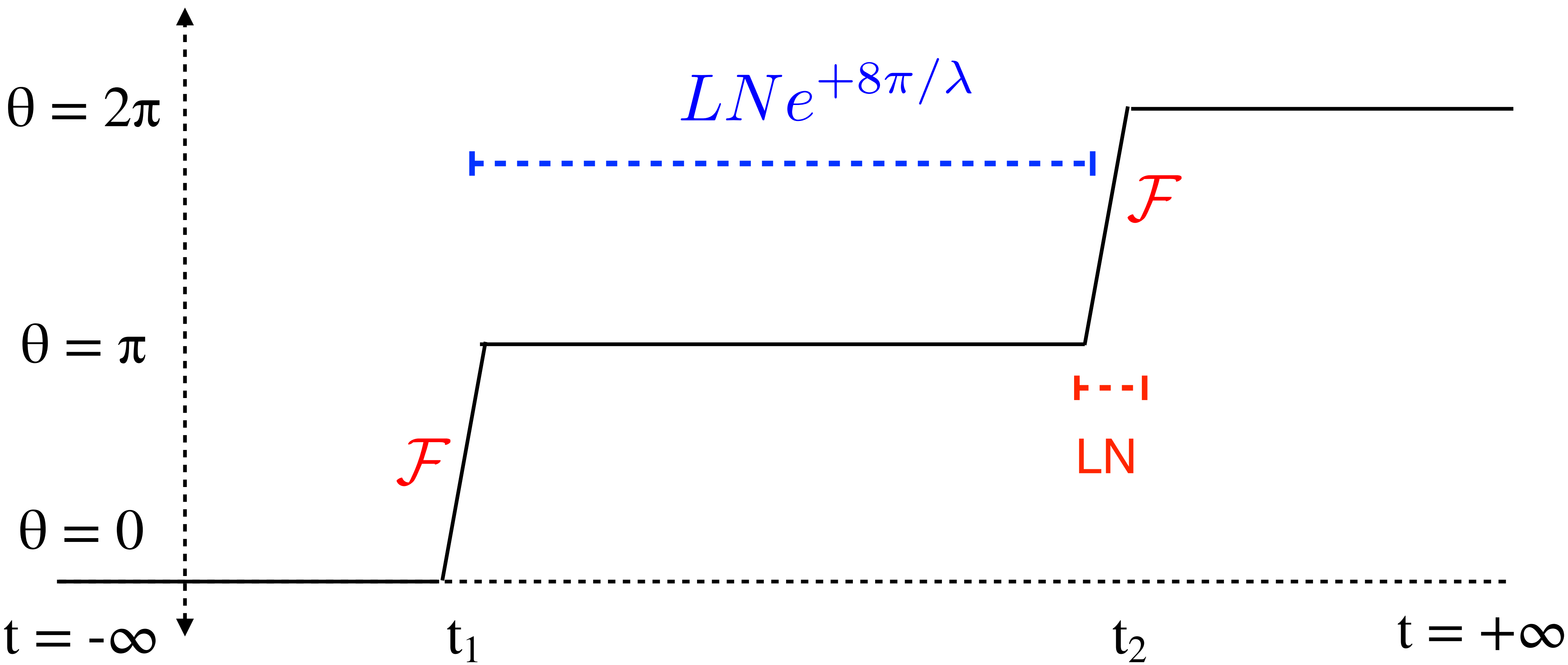
KK fractons in PCM appear same way as KK monopoles in compactified YM theories with non-trivial Wilson lines

The sum over finite-action configurations

$$\langle \mathcal{O}(\lambda) \rangle = \sum_{n=0}^{\infty} p_{0,n} \lambda^n + \sum_c e^{-S_c/\lambda} \sum_{k=0}^{\infty} p_{c,n} \lambda^n$$

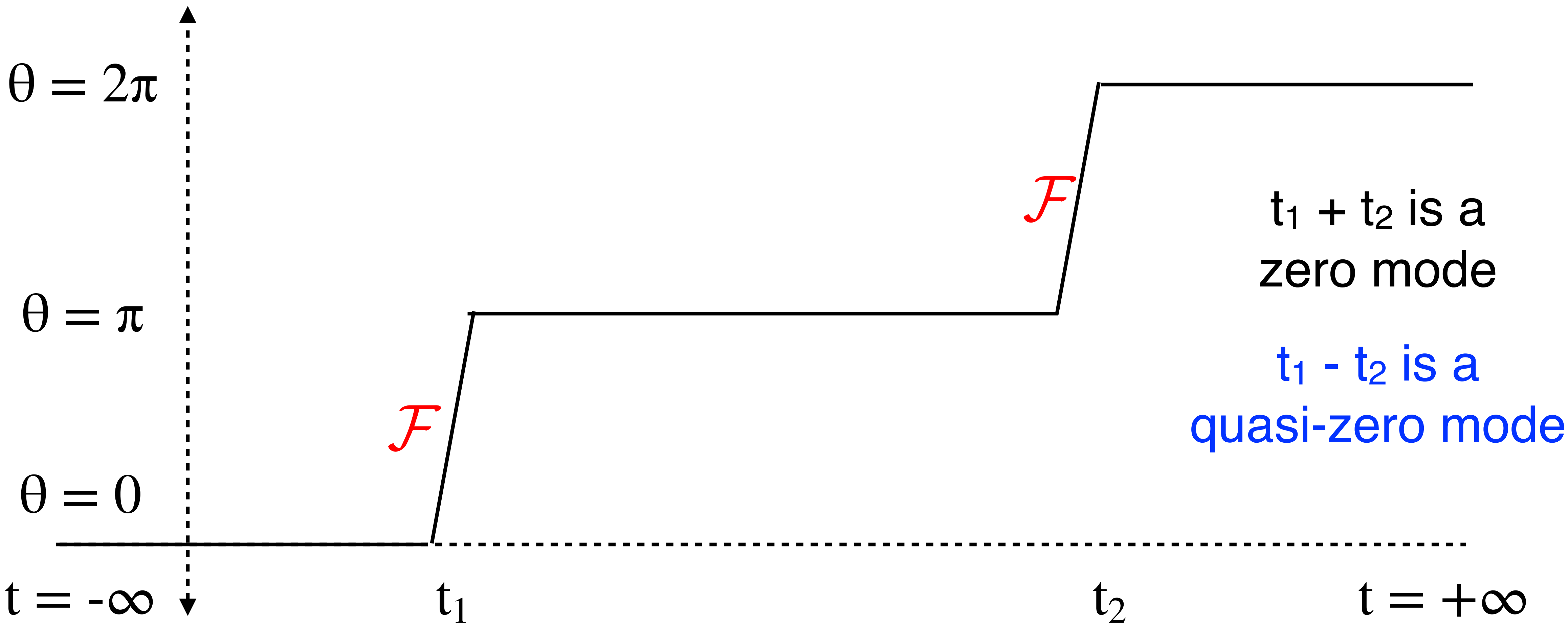
How can NP saddles give ambiguous contributions to path integral?

Small-L theory weakly coupled, dilute fracton gas approximation is valid



This is the typical separation between tunneling events

Contribution from fracton-fracton events



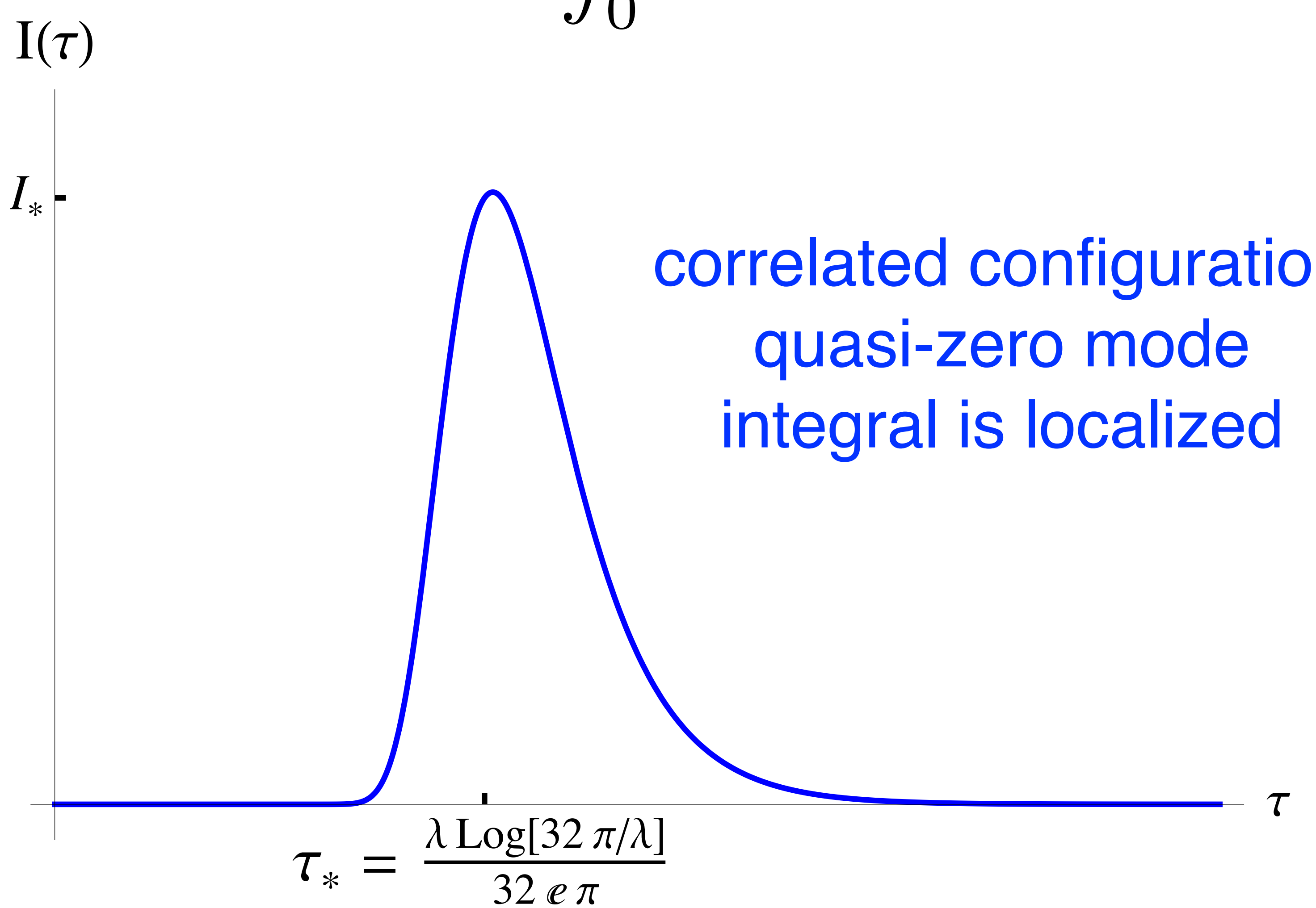
Action depends on $t_1 - t_2$, but only exponentially weakly for large $t_1 - t_2$.

Pairs of fractons are quasi-saddle-point!

Existence of quasi-zero fluctuation modes gives rise to existence of correlated tunneling events

Correlated fracton-fracton events

$$I_{\mathcal{F}\mathcal{F}} \sim e^{-2S_F} \int_0^\infty d\tau \, \tau \, e^{-\left(\frac{32\pi}{\lambda} e^{-\tau} + \tau\right)}$$



$\tau = t_1 - t_2$

Scale separation

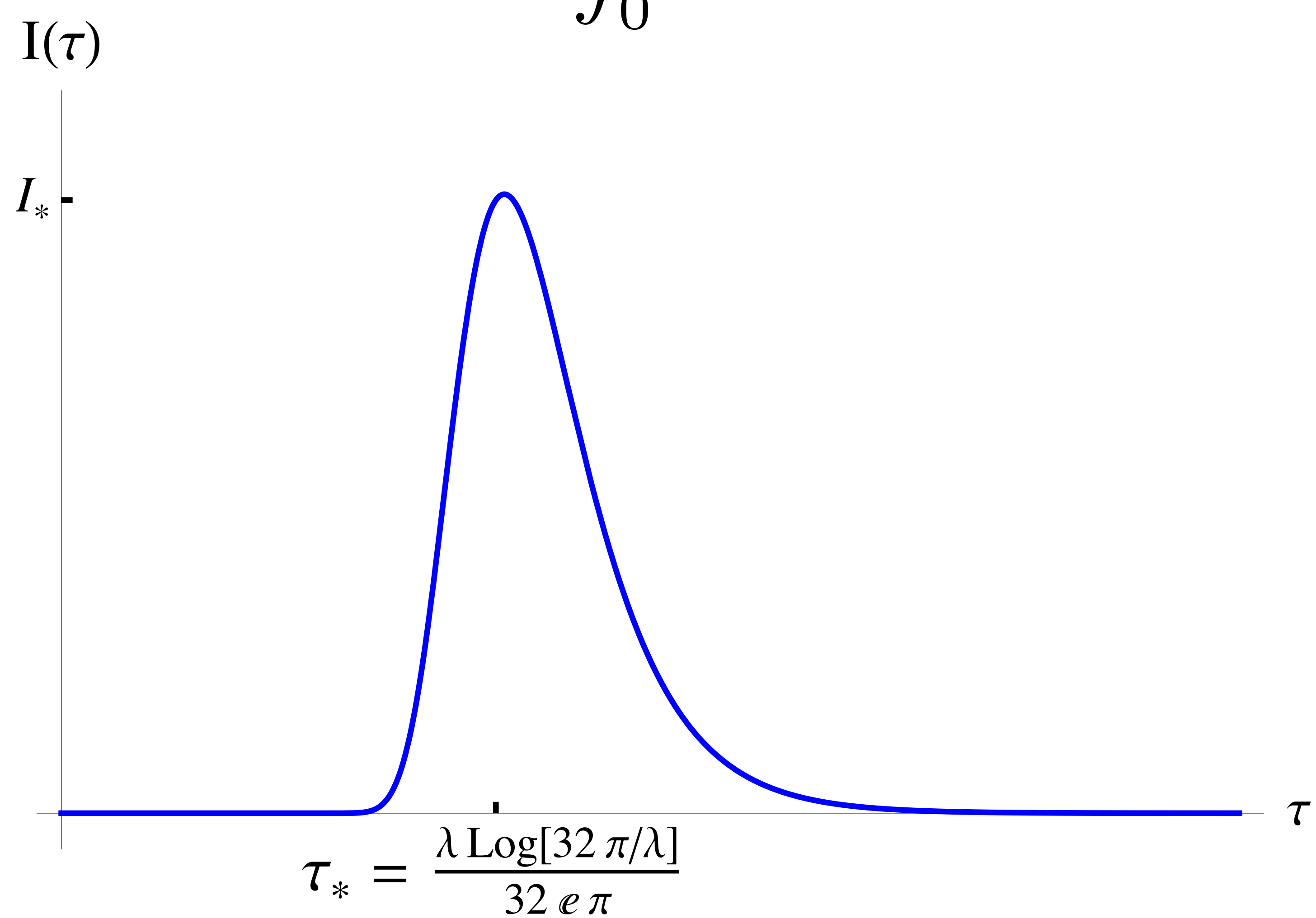
$$LN \ll LN \log\left(\frac{32\pi}{\lambda}\right) \ll LN e^{+8\pi/\lambda}$$

Fracton gas is dilute!

fracton 1-event | correlated 2-event - a ‘bion’ | uncorrelated 2-event

Correlated fracton-fracton events

$$I_{\mathcal{F}\mathcal{F}} \sim e^{-2S_F} \int_0^\infty d\tau \tau e^{-\left(\frac{32\pi}{\lambda} e^{-\tau} + \tau\right)}$$



$\tau = t_1 - t_2$

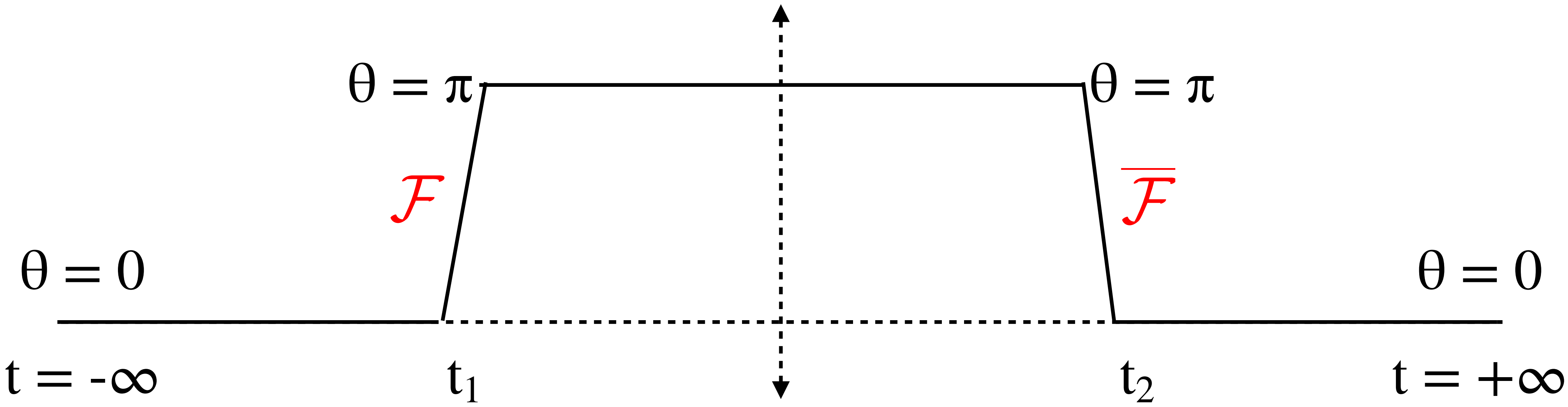
amplitude:

$$[\mathcal{F}\mathcal{F}] = \left(-\log\left[\frac{32\pi}{\lambda}\right] - \gamma\right) \frac{16}{\lambda} e^{-2S_F}$$

Correlated fracton-fracton events are unambiguous

Contribution from fracton-anti-fracton events

Sometimes there are events which involve tunneling back and forth



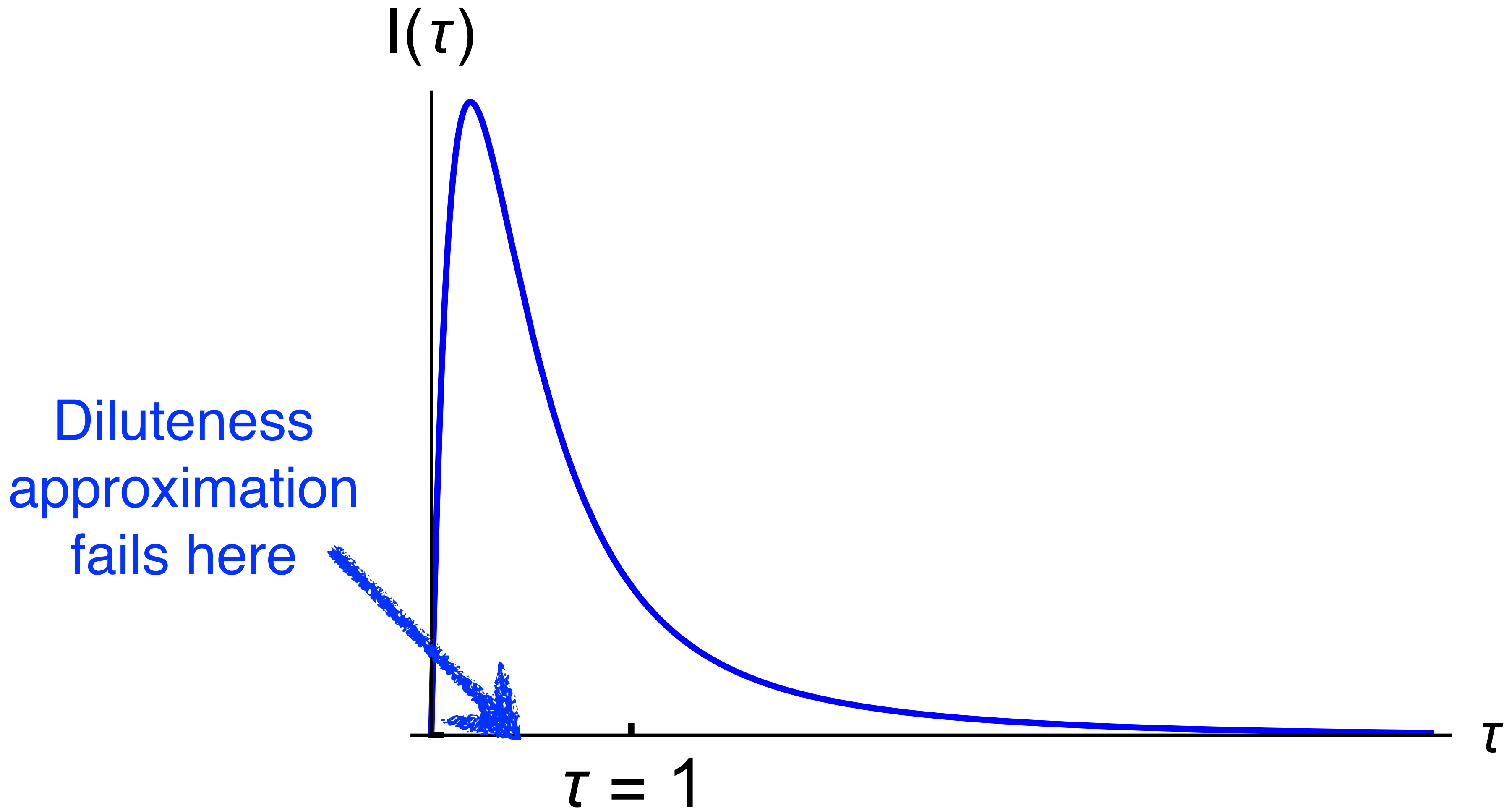
t_1 - t_2 is still a quasi-zero-mode

Treating the quasi-zero modes for fracton-anti-fracton events is subtle. This is where the ambiguities live!

Trouble with fracton-anti-fracton events

The anti-fracton-fracton interaction is attractive!

$$I_{\mathcal{F}\bar{\mathcal{F}}} \sim e^{-2S_F} \int_0^\infty d\tau \tau e^{-\left(-1 \times \frac{32\pi}{\lambda} e^{-\tau} + \tau\right)}$$



So makes no sense to take integral as is, fracton-anti-fracton configurations only make sense for large **τ**!

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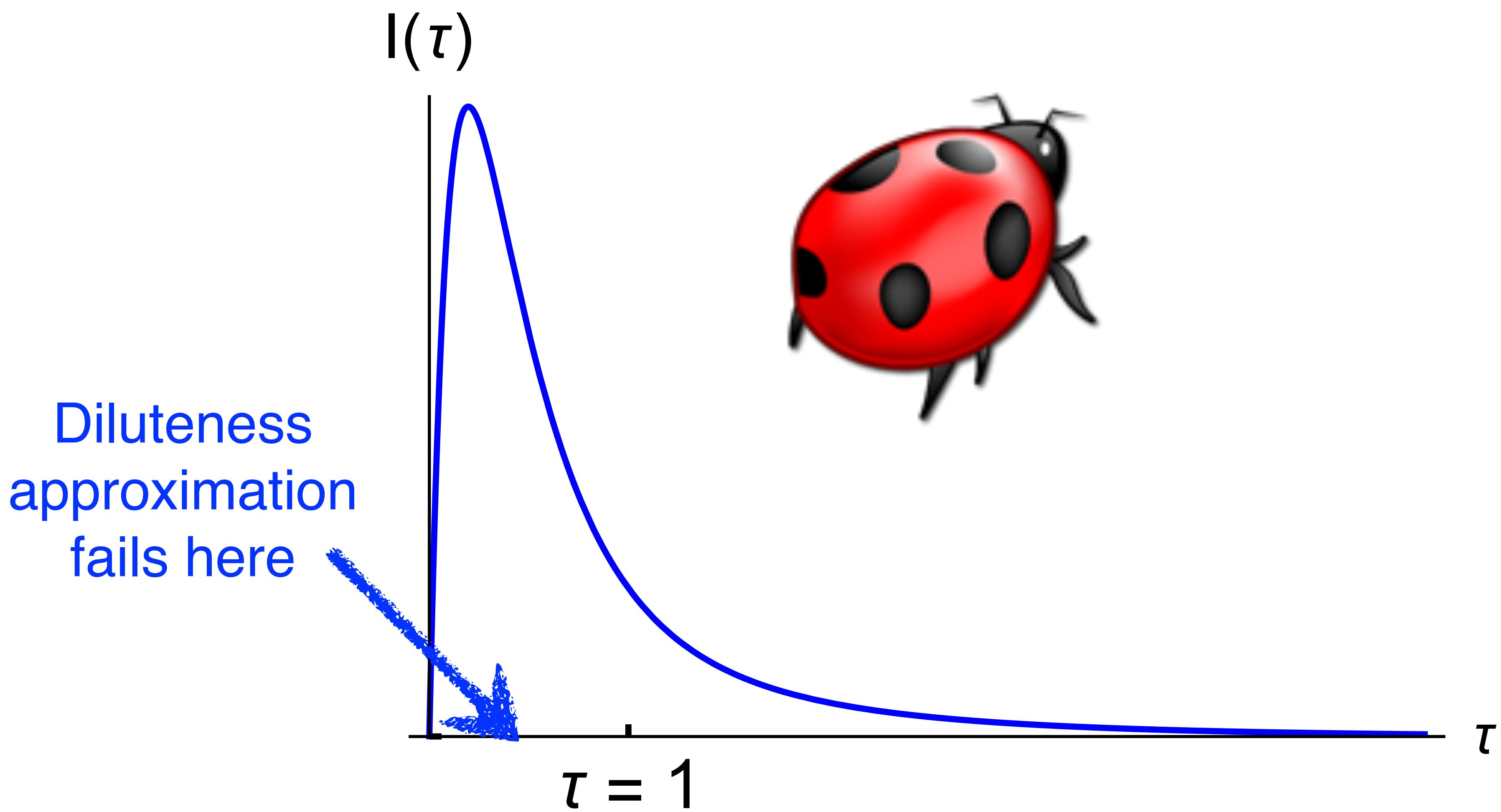

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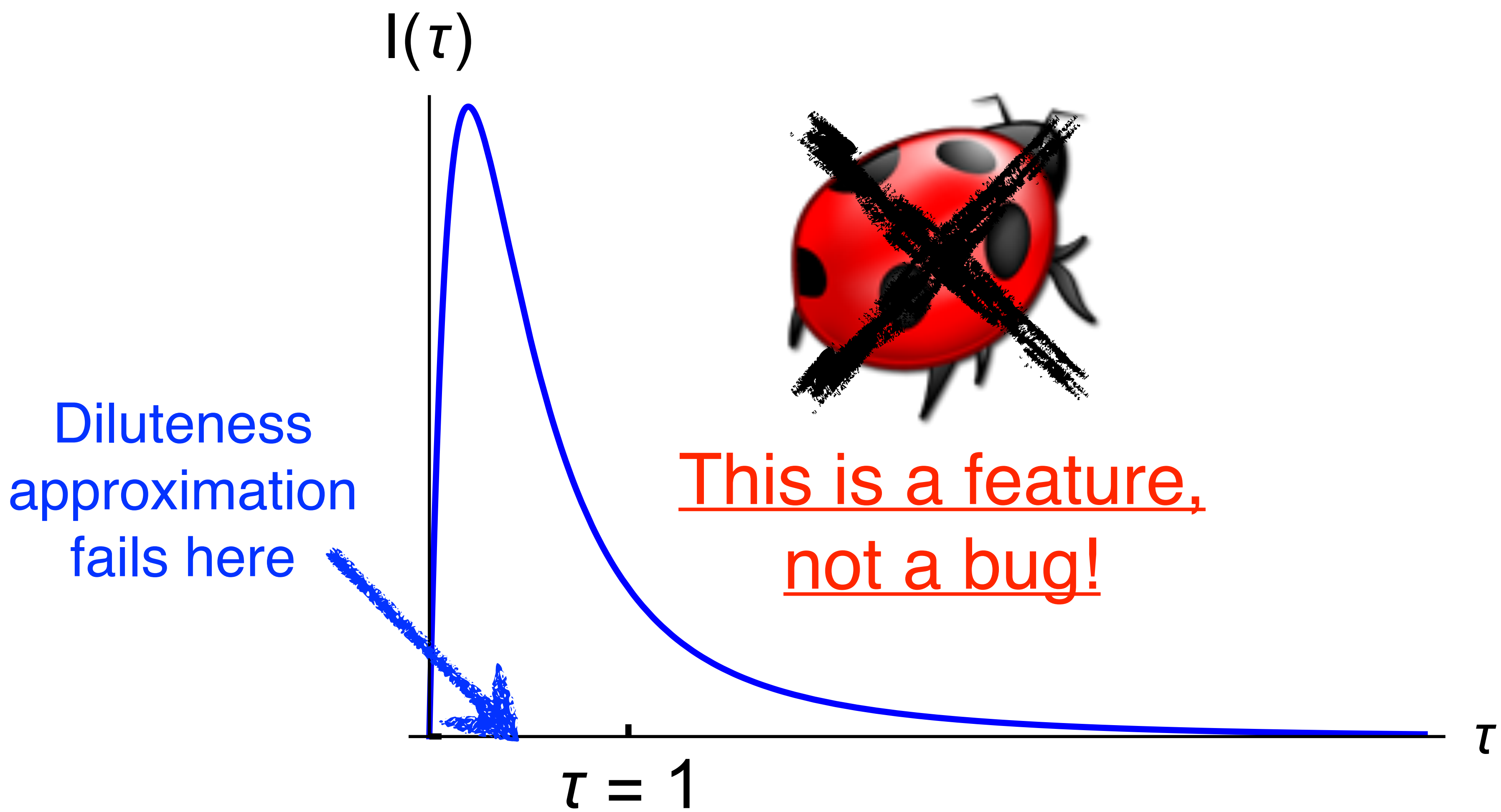


This is a feature, not a bug!

So makes no sense to take integral as is, fracton-anti-fracton configurations only make sense for large **τ**!

Making sense of fracton-anti-fracton events

Decomposition of path integrals integration cycle into the basis of Lefshetz thimbles appears to geometrize resurgent transseries.

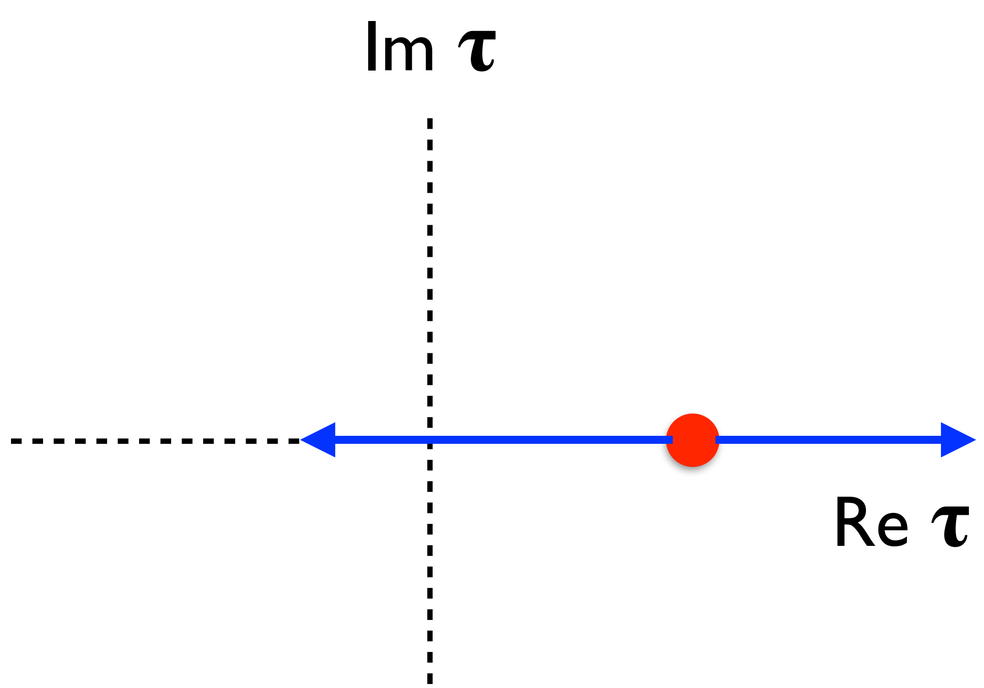
Each 'thimble' is a steepest-descent manifold attached to a critical point

Pham 1983; Witten 2010

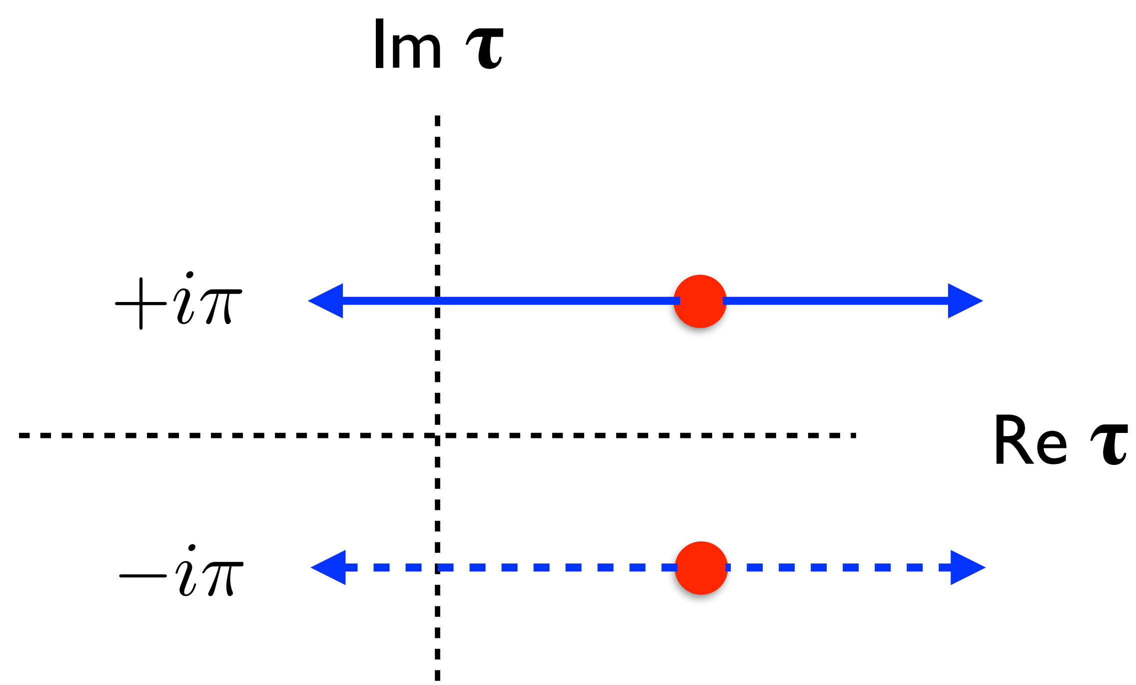
Systematically integrating over the quasi-zero mode turns out to need this machinery

Basar, AC, Dunne, Dorigoni, Unsal, coming soon

Fracton-fracton integration cycle



Fracton-anti-fracton integration cycle(s)



Contribution of fracton-anti-fracton events

Systematically integrating over the quasi-zero mode turns out to need this machinery

Basar, AC, Dunne, Dorigoni, Unsal, coming soon

Fracton-anti-fracton events give an ambiguous contribution:

$$[\mathcal{F}\bar{\mathcal{F}}] = -\left(-\log\left[\frac{32\pi}{\lambda}\right] - \gamma_E\right)\frac{16}{\lambda}e^{-16\pi/\lambda} \pm i\frac{16\pi}{\lambda}e^{-16\pi/\lambda}$$

(More precisely, the contribution depends on arg $\lambda = 0^{\pm}$, as does perturbation theory.)

Cancellation of ambiguities

Only the sum of P and NP contributions is physical.

If QFT observables are actually resurgent transseries, resurgence theory predicts:

$$\mathrm{Im}\left[\mathcal{S}_{\pm}\mathcal{E}(g^2) + [\mathcal{F}\bar{\mathcal{F}}]_{\pm}\right] = 0, \text{ up to } \mathcal{O}\left(e^{-4S_F}\right)$$

Preceding results implies that this works in PCM

Leading renormalon ambiguities of perturbation theory cancel against ambiguities in saddle-point sum

Illustrates that exact information about NP physics is present in perturbation theory, albeit in coded form!

What we learned so far…

Even when there's no topology, resurgence predicts existence of NP saddle points with specific properties, which can then be found.

In semiclassical domain, renormalon ambiguities systematically cancel against contributions of non-BPS NP saddles

Renormalons closely related to mass gap, as 't Hooft dreamt

Fracton tunneling events give rise to mass gap of the model…

… and fracton-anti-fracton events give rise to the renormalons

All results so far fit conjecture of resurgent nature of observables in QFTs with weak-coupling limits

Many applications seem likely

Resurgence theory yields major new insights into the non-perturbative structure of quantum field theories

Vital to explore relations to analytic continuation of path integrals

May get better understanding of QFTs with complex actions

Resurgence theory and Lefshetz thimble technology play vital role in seeing how instantons appear in real-time Feynman path integrals. AC, Unsal 2014

Improved understanding of connections between strong and weak coupling regimes AC, Koroteev, Unsal 2014

Applications of resurgence in SUSY QFTs Aniceto, Russo, Schiappa 2014; Basar, Dunne 2015

The work has just begun. Lots of fun things to do!