General Game Theory Model

p_i = frequency of strategy i

g_{ij} = payoff of i against j

Mean payoff of strategy i:

$$G_i = \sum_j p_j g_{ij}$$

Mean payoff of whole population:

$$\bar{G} = \sum_i \sum_j p_i p_j g_{ij}$$

Deterministic Evolutionary Dynamics

$$\frac{dp_i}{dt} = p_i (G_i - \bar{G})$$

cf. quasispecies theory
with no mutation

"replicator equation"

Can strategy j invade a population of i?

Let $p_j = \varepsilon$ (small) $\qquad p_i = 1 - \varepsilon$

$$G_i = (1-\varepsilon) g_{ii} + \varepsilon g_{ij}$$
$$G_j = (1-\varepsilon) g_{ji} + \varepsilon g_{jj}$$

$G_j > G_i$ if $\quad g_{ji} > g_{ii}$

OR $\quad g_{ji} = g_{ii}$ and $g_{jj} > g_{ij}$

Strategy i is an ESS if there is no j that can invade

i.e. **for all** j $\quad g_{ii} > g_{ji}$

OR $\quad g_{ii} = g_{ji}$ and $g_{ij} > g_{jj}$

Evolutionary Game Theory

The Hawk – Dove game

Aggression – aggressive displays, bluff

Players compete for resource of value $\underline{V}$
(food, territory, mate ...)

Cost $\underline{C}$ associated with fight.
(injury, time, energy ...)

Payoff to first player

1st player \ 2nd player	H	D
H	$\frac{1}{2}(V-C)$	V
D	0	$V/2$

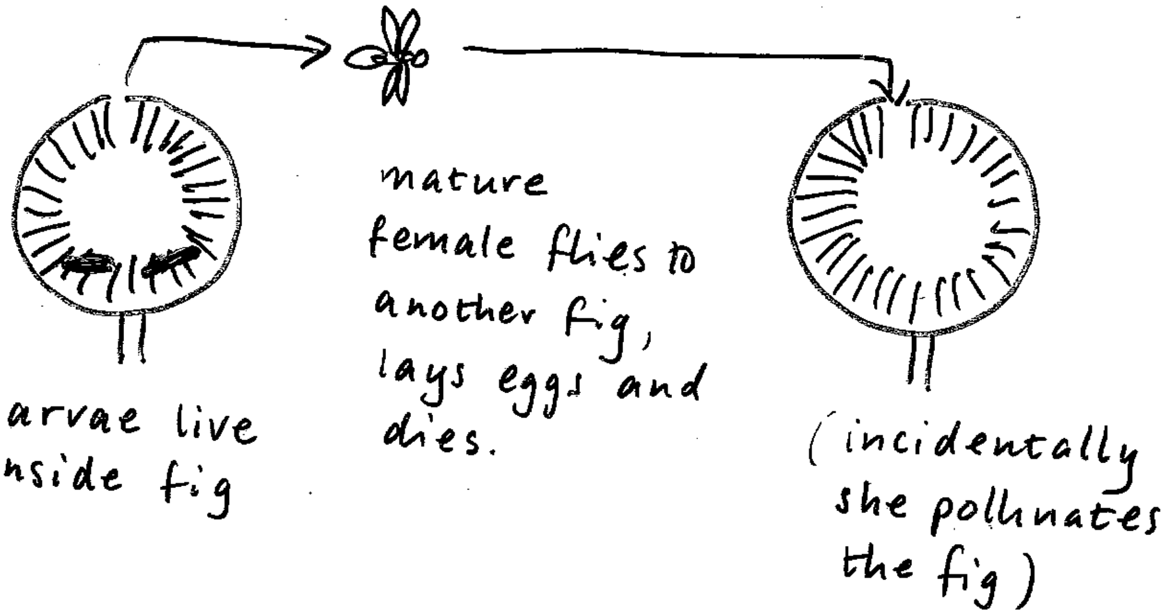
If $V > C$ always best to be an H

H gets higher payoff
H multiplies faster and takes over population

If $V < C$ best to be H if opponent is D
" " " D " " " H

Population evolves towards a mixture with

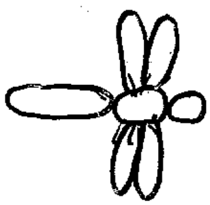
Fig Wasps



Polymorphism observed : 2 types of males

Jawed males  = H

Fight to mate with females before they leave the fig

Winged males  = D

Only mate with females outside fig.

V = mating

C = injury or death

V must be less than C

Bourgeois Game - ownership

assymetric contests

3 strategies H, D, B

B behaves like H if owner and like D if intruder.

if $V > C$ H is ESS

if $V < C$ B is ESS (owner always wins)

Note that the B ESS maximises population payoff, whereas in the HD game the ESS does not maximise payoff.

Assymetries can be used to settle contest with no cost

General model

2 or more strategies with payoff matrix specified.

Calculate payoff of each strategy — depends on frequencies of other strategies in the population.

If payoff is higher than average, frequency ↑

" " " lower " " , " ↓

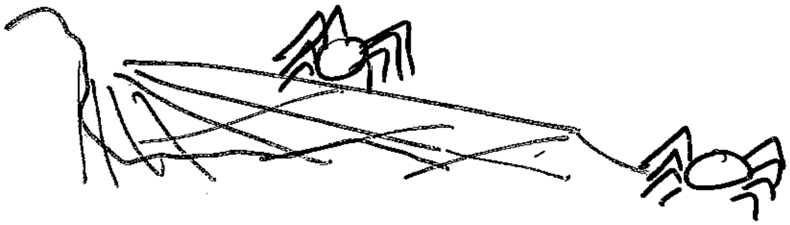
Dynamics leads to stationary states called Evolutionarily Stable Strategies

An ESS is one which cannot be invaded by any other strategy.

ESSs can be pure or mixed (ie. probabilistic)

Funnel web spiders

Riechert (1979)



Fight to own web sites in favoured places

Fights between owner and intruder.

Scale of aggression observed:

threat < web shaking < contact < biting/tumbling

Spiders may choose to withdraw or escalate.

High levels of aggression have significant energy cost and risk of injury.

Many contests observed - measured levels of aggression and duration

Results:

1. Fights longer and more aggressive when good sites are rare.
2. Assymmetries affect outcome.
   - owner more likely to win
   - larger individual more likely to win.
3. Contests longer if individuals of similar size
4. Longest contests if web of high value and owner slightly smaller than intruder

War of Attrition

Value V goes to player who waits longest.
Cost = duration of contest.

A waits up to t_A
B waits up to t_B

	Payoff to A	Payoff to B
$t_A > t_B$	$V - t_B$	$-t_B$
$t_B > t_A$	$-t_A$	$V - t_A$
$t_A = t_B$	$\frac{V}{2} - t_A$	$\frac{V}{2} - t_A$

ESS : choose time randomly from an exponential distribution

$$P(t) = \frac{1}{V} e^{-t/V}$$

Possible Example - Dung Flies

Parker + Thompson (1980)

Females lay eggs on cow pats
Males wait on fresh cowpats for females to arrive and mate

Exponential waiting time observed
Mating success found to be independent of waiting time on average.
Suggests a mixed ESS.

Population Genetics v. Game Theory

Dynamics assumes like begets like.
(Asexual with no mutation ....?)

Should specify genetic system:

eg. single locus diploid system with dominanc

AA } Hawk
Aa }

aa - Dove

Should specify reproductive system:

sexual or asexual

mutations
recombination

multiple matings / family size etc.

But - Often genetic system is complex

Often we don't know it

Often is doesn't matter.